Revised as per HEC New UEP 2023

SHAHEED BENAZIR BHUTTO WOMEN UNIVERSITY PESHAWAR

DEPARTMENT OF MATHEMATICS

CURRICULUM
SESSION 2023-ONWARDS

CURRICULUM
2023-ONWARDS

NOTE: Approved from the 8th meeting of BOS, 12th meeting of BOF, 19th meeting of Academic Council and 49th meeting of Syndicate

Department Curriculum Committee

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Dean Faculty of Sciences & Social Science,
SBBWU

DEPARTMENT OF MATHEMATICS

INTRODUCTION

The Department of Mathematics at Shaheed Benazir Bhutto University, Peshawar was established in the year 2005. Department of Mathematics is one of the pioneer departments at SBBWU. The department offers different degree programs, such as BS, MPhil and PhD both in pure and applied mathematics. About 80 percent of the full time faculty at the department are PhD degree holders.

VISION STATEMENT OF DEPARTEMT

To be a leading Department in both public and private sector Universities in research centered teaching and funded projects, to equip female students with in-depth knowledge of how to use mathematics tools to solve the real world problems so that they can be a useful citizen of Pakistan and have benefit to this world.

MISSION STATEMENT OF DEPARTMENT

Department of Mathematics access to basic and higher education opportunities in mathematics through education, outstanding research, scholarships and practice that enable students to develop knowledge and skills necessary to achieve their professional goals

BS-PROGRAM

VISION STATEMENT OF PROGRAM

The vision of the program is to impart a quality education and productive research in different fields of Mathematics. It focuses to produce highly skilled professionals, scholars and researchers to meet the contemporary and future challenges.

MISSION STATEMENT OF PROGRAM

The mission of the BS program in the Department of Mathematics is to provide students with an understanding of mathematical thought and knowledge; the ability to use this understanding to produce and communicate mathematics; and the preparation to apply these skills in advanced degree programs and/or careers requiring expertise in mathematics.

ELIGIBILITY CRITERIA

- F.A/F. Sc with Mathematics
- 45% marks in aggregate

STRUCTURE

Sr.	Categories	Credit Hours Min – Max
1.	General Education (Gen Edu) Requirements: Mandatory Courses of General Education.	30 – 30
2.	Major (Disciplinary) Requirements: Area of Study in Which the Degree is offered	*72 ≥
3.	Interdisciplinary/Allied Requirements (To Support Horizon of the Major)	**12 ≥
4.	Field Experience/Internship (Practical Work Experience related to a Student's Field of Study or Career interest)	03 – 03
5.	Capstone Project or Capstone Research Project	03 – 06
	Total	**120 – 144**

***The Credit Hours for the courses of Major Disciplines may vary but not less than 72 Credit Hours.**
****The Credit Hours for Interdisciplinary/Allied Courses may vary but not less than 12 Credit Hours.**

- Total number of Credit hours: 120-144
- Duration: 4 years
- Semester duration: 16-18 weeks
- Semesters: 8
- Course Load per Semester: 15-18 Cr hr
- Number of courses per semester: 4-6 (not more than 3 lab / practical courses)

SCHEME OF STUDIES OF BS -_MATHEMATICS_4-YEAR PROGRAM (SESSION 2023 & Onwards)

Semester	Category	Course Codes	Course Title	Lectures	Lab	Cr.Hrs
Semester 1	Art & Humanities	AH-301	Art & Hum	2	0	2
	Islamic Studies/Religious Studies/Ethics	ISL-301	Islamic Studies/Religious Studies	2	0	2
	Interdisciplinary/Allied	PHY-301	Mechanics	2	1	3
	Functional English	ENG-303	Functional English	3	0	3
	Major I	MTH-311	Calculus-I	4	0	4
	Major II	MTH-312	Elements of Set Theory and Mathematical logic	3	0	3
			Total			**17**
Semester 2	Social Sciences	000	Social Science	2	0	2
	Expository Writing	ENG-304	Expository Writing	3	0	3
	Interdisciplinary/Allied	PHY-307	Introductory Electricity & Magnetism	2	1	3
	Ideology and Constitution of Pakistan	PST-313	Ideology and Constitution of Pakistan	2	0	2
	Major III	MTH-321	Calculus-II	4	0	4
	Major IV	MTH-322	Discrete Mathematics	3	0	3
			Total			**17**
Semester 3	Quantitative Reasoning (QR I)	MTH-401	Quantitative Reasoning-I (QR-I)	**3**	**0**	**3**
	Application of Information and Communication Technologies.	CSC-308	Application of Information and Communication Technologies	2	1	3
	Natural Science	000	Natural Science	2	1	3
	Entrepreneurship	MS-309	Introduction to Entrepreneurship	2	0	2
	Major V	MTH-411	Calculus-III	4	0	4
	Major VI*	MTH-412	Group Theory	3	0	3
			Total			**18**

Semester 4	Civic and Community Engagement	PSC-418	Civic and Community Engagement	2	0	3
	Quantitative Reasoning (QR II)	MTH-402	Quantitative Reasoning-II (QR-II)	3	0	3
	Major VII	MTH-421	Computing Tools	2	1	3
	Major VIII	MTH-422	Linear Algebra	3	0	3
	Major IX	MTH-423	Number Theory	3	0	3
	Interdisciplinary/Allied Course	STAT-403	Probability Theory	3	0	3
			Total			**18**
Semester 5	Major X	MTH-511	Topology	3	0	3
	Major XI	MTH-512	Complex Analysis	3	0	3
	Major XII	MTH-513	Ordinary Differential Equations	3	0	3
	Major XIII	MTH-514	Real Analysis-I	3	0	3
	Major XIV	MTH-515	Numerical Analysis	3	0	3
			Total			**15**
Semester 6	Interdisciplinary/Allied Course	PHY-513	Classical Mechanics	3	0	3
	Major XV	MTH-521	Partial Differential Equations	3	0	3
	Major XVI	MTH-522	Real Analysis-II	3	0	3
	Major XVII	MTH-523	Rings and Fields	3	0	3
	Major XVIII	MTH-524	Affine and Euclidean Geometry	3	0	3
			Total			**15**
Semester 7	Internship (Mandatory)	MTH-698	Internship (Mandatory)	0	3	3
	Major XIX	MTH-611	Measure Theory-I	3	0	3
	Major XX	MTH-612	Mathematical Methods	3	0	3
	Major XXI(Elective)	MTH-613	Graph Theory	3	0	3
	Major XXII	MTH-614	Integral Equations	3	0	3
	Capstone Research Project OR Capstone Project + Optional Course	MTH-699/697	Capstone Research Project OR Capstone Project + Optional Course	0	3	3
			Total			**18**
Semester 8	Major XXIII(Elective)	MTH-621	Fluid Mechanics	3	0	3
	Major XXIV	MTH-622	Measure Theory-II	3	0	3
	Major XXV	MTH-623	Mathematical Statistics	3	0	3
	Major XXVI	MTH-624	Functional Analysis	3	0	3
	Capstone Research Project OR Capstone Project + Optional Course	MTH-699/697	Capstone Research Project OR Capstone Project + Optional Course	0	3	3
			Total			**15**

List of Optional/ Elective Courses for BS- Mathematics

S.no	Course Title	Course Codes

1.	Differential Geometry	MTH-631
2.	Optimization Theory	MTH-632
3	Algebraic Topology	MTH-633
4.	Galois Theory	MTH-634
5.	Riemannian Geometry	MTH-635
6.	General Relativity	MTH-636
7.	Dynamical Systems	MTH-637
8.	Quantum Mechanics	MTH-638
9.	Lie Groups and Lie Algebra	MTH-639
10.	Special Relativity	MTH-640
11.	Convex Analysis	MTH-641
12.	Dynamics	MTH-642
13.	Computational Fluid Dynamics	MTH-643
14.	Mathematical Modeling and Simulations	MTH-644
15.	Vector Analysis	MTH-645

NOTE: Semester wise course contents of all the courses Major/Elective are given in ANNEXTURE-A

INTERDISIPLINARY/ALLIED COURSES		
Coe Code	**Course Title**	**Credit Hours**
MTH-301	Calculus & Analytical Geometry	3(3,0)
MTH-302	Advanced Calculus	3(3,0)
MTH-404	Multivariate Calculus	3(3,0)
MTH-405	Linear Algebra	3(3,0)
MTH-501	Differential Equations	3(3,0)]
MTH-502	Computational Mathematics	3(3,0)
MTH-303	Geometry and Drafting	3(3,0)
MTH-304	Business Mathematics	3(3,0)
MTH-503	Teaching of Mathematics (Professional)	3(3,0)
MTH-504	Content Course I (From Selected Discipline I)	3(3,0)
MTH-601	Content Course II (From Selected Discipline I	3(3,0)
	MANDATORY COURSES	
MTH-401	Quantitative Reasoning-I(QR-I)	3(3,0)
MTH-402	Quantitative Reasoning –II(QR-II)	3(3,0)

NOTE: The contents of all the Allied and Mandatory courses are given in ANNEXTURE-B

MPHIL AND PHD PROGRAM

VISION STATEMENT OF PROGRAM

The vision of the program is to impart a quality education and productive research in different fields of Mathematics. It focuses to produce highly skilled professionals and researchers to meet the contemporary and future challenges.

MISSION STATEMENT OF PROGRAM

The mission of the MPhil/ PhD program in the Department of Mathematics is to equip students with advanced mathematical knowledge that may be used in different fields and technology and to prepare them for productive careers in research and education .

ELIGIBILITY CRITERIA

MPHIL PROGRAM:

- Relavent Masters degree/4years eduction after intermediate(130 credit hours). Minimum 2.5 CGPA/ 2nd division in Annual System.
- Candidate must have passed Departmental test/GAT general test conducted by NTS with at least 50% marks.

PHD PROGRAM:

- Candidates having MPhil 18 years of education with a minimum of 0.3 CGPA in semester system or 1st division in Annual system.
- Candidate must have passes the departmental test/ GAT subject test conducted by NTS with at least 60% marks.

NOTE: LIST AND CONTENTS OF ALIGNED COURSES OF MPHIL AND PHD ARE GIVEN IN ANNEXTURE-C

ANNEXTURE-A

COURSE CONTENTS OF MAJOR AND ELECTIVE COURSES FOR BS-MATHEMATICS(SESSION: 2023-ONWARDS)

<table>
<tr><td colspan="2">SEMESTER-I</td></tr>
<tr><td>Course Name: Calculus-I</td><td>Course Code:MTH-311</td></tr>
<tr><td>Course Structure: Lectures: 4, Lab:0</td><td>Credit Hours: 4</td></tr>
<tr><td colspan="2">Prerequisites: Knowledge of Intermediate Calculus</td></tr>
<tr><td colspan="2"><u>Specific Objectives of course:</u>
• Calculus serves as the foundation of advanced subjects in all areas of mathematics.
• The objective of this course is to introduce students to the fundamental concepts of limit, continuity, differential and integral calculus of functions of one variable.

<u>Course Outline:</u>

Functions and graphs: Domain and range of a function. Examples: polynomial, rational, piecewise defined functions, absolute value functions, and evaluation of such functions. Operations with functions: sum, product, quotient and composition. Graphs of functions: linear, quadratic, piecewise defined functions.Lines and systems of equations: Equation of a straight line, slope and intercept of a line, parallel and perpendicular lines. Systems of linear equations, solution of system of linear equations. Nonlinear systems: at least one quadratic equation. Limits and continuity: Functions, limit of a function. Graphical approach. Properties of limits. Theorems of limits. Limits of polynomials, rational and transcendental functions. Limits at infinity, infinite limits, one-sided limits. Continuity.Derivatives: Definition, techniques of differentiation. Derivatives of polynomials and rational, exponential, logarithmic and trigonometric functions. The chain rule. Implicit differentiation. Rates of change in natural and social sciences. Related rates. Linear approximations and differentials. Higher derivatives,</td></tr>
</table>

Leibnitz's theorem.**Applications of derivatives:** Increasing and decreasing functions. Relative extrema and optimization. First derivative test for relative extrema. Convexity and point of inflection. The second derivative test for extrema. Curve sketching. Mean value theorems. Indeterminate forms and L'Hopitals rule. Inverse functions and their derivatives.

Learning Outcomes:

Upon successful completion of this course, students will:

Determine whether a function is continuous and/or differentiable at a point using limits.

- Draw graphs of algebraic and transcendental functions considering limits, continuity, and differentiability at a point.
- Use differentiation rules to differentiate algebraic and transcendental functions.
- Identify appropriate calculus concepts and techniques to provide mathematical models of real-world situations and determine solutions to applied problems.

Recommended Books:

1. Anton, H., Bivens, I. C., & Davis, S. (2021). *Calculus*. John Wiley & Sons.
2. Bressoud, D., Ghedamsi, I., Martinez-Luaces, V., & Törner, G. (2016). *Teaching and learning of calculus*. Springer Nature.
3. Stewart, J. (2018). *Single variable calculus: Concepts and contexts*. Cengage Learning.
4. Salas, S. L., Etgen, G. J., & Hille, E. (2021). *Calculus: One and several variables*. John Wiley & Sons.
5. Stewart, J., Clegg, D. K., & Watson, S. (2020). *Calculus: early transcendentals*. Cengage Learning.
6. Adams, R. A., & Essex, C. (2018). *Calculus: a complete course*. Pearson.

Course Title: Art & Humanities	**Course Code: 000**
Course Structure: Lectures: 3, Labs: 0	**Credit Hours: 2**
Prerequisites: Nil	
Course Objective:	
Course Outline: **The Course contents will be taken from the booklet of mandatory and general courses.**	
Course Outcomes:	

Recommended Books: Latest Edition of the Following Books.

Course Title: Islamic Studies/Religious Studies/Ethics	**Course Code: ISL-301**
Course Structure: Lectures: 2, Labs: 0	**Credit Hours: 2**
Prerequisites: Nil	
Course Objective: This course is designed to provide students with a comprehensive overview of the fundamental aspect of Islam, its beliefs practices History and influence on society. It will further familiarize the students with a solid foundation in understanding Islam from an academic and cultural perspective. Through this course students will have and enhanced understating of Islam's multifaceted dimensions which will enable them to navigate complex discussions about Islam's Historical and contemporary role fostering empathy respect and informed dialogue.	
Course Outline: **Introduction to Islam:** Definition of Islam and its core beliefs The Holy Qur'an (Introduction, Revelation and compilation, Hadith and Sunnah (Compilation Classification and Significance) Key theological concepts and themes (Tawhid , Prophet hood Akhirah etc., **Seerat of Holy Prophet (S.A.W)** Life and legacy of the Holy prophet **(S.A.W** Diverse role of the Holy Prophet (as an individual, educator, peace maker, leader etc.**), Islamic History and civilization** World Before Islam Rashidun Caliphate and expansion of Islamic rule, Contribution of Muslim scientists and philosophers in shaping world civilization, **Islamic Jurisprudence: (Fiqh)** Fundamental Sources of Islamic Jurisprudence Pillars of Islam and their significance Major Schools of Islamic Jurisprudence**,** Significance and principles of Ijtihad, **Family and Society in Islam** Status and rights of woman in Islamic Teachings, Marriage, Family, and gender roles in Muslim society, Family structure and values Muslim society, **I**slam & the Modern World.	
Course Outcomes: After completing the course, the students will be able to 1. Demonstrate enhanced knowledge of Islamic foundational beliefs, practices historical development spiritual values and ethical principles 2. Describe basic source of Islamic law and their application in daily life	

Identify and discuss contemporary issue being faced by the Muslims world including social challenges, gender role and interfaith interactions

Recommended Books: Latest Edition of the Following Books.

Suggested Instructional Materials

1. The five Pillars of Islam: A journey thought the Divene Acts of Worship by Muhammad Mustafa Al Azami
2. The Five Pillars of Islam: A Framework for Islamic Values and Character Building by Musharraf Hussain
3. Towards Understanding Islam By Abul, A' la Mawdudi
4. Islami Nazria e Hayat by Khurshid Ahmad
5. An Introduction to Islamic theology by John Rearard
6. Islamic Civilization Foundations Belief and Principles by Abul A la Mawdudi
7. Women and Social Justices an Islamic Paradigm by Dr Anis Ahmad
8. Islam its Meaning and Message "By Khushid Ahmad

Course Name: Mechanics	**Course Code: PHY-301**
Course Structure: Lectures: 2, Labs: 1	**Credit Hours:** 3
Prerequisites: None	

Course Objectives:

Mechanism design course would help students in achieving the following objectives:

- To help in understanding different types of mechanisms and underlying concepts of kinematics.
- To build mathematical models of mechanisms and develop position, velocity, and acceleration relations. To understand the dynamic characteristics of planar mechanism.

Course Outline: Basic Concepts: Units and Dimensions, SI Units, Changing Units, Scalars and Vectors, Adding Vectors: Graphical as well as Component Method, Multiplying Vectors: Dot and Cross Products. Motion in One, Two and Three Dimensions: Position and displacement, Velocity and Acceleration, Motion under Constant Acceleration, Projectile Motion, Uniform Circular Motion, Relative Velocity and Acceleration in One and Two Dimensions, Inertial and Non-Inertial Reference Frames. Newton's Laws: Newton's Laws of Motion and their Applications involving some particular forces including Weight, Normal Force, Tension, Friction, and Centripetal Force, Newton's Law of Gravitation, Gravitational Potential Energy, Escape Velocity, Kepler's Laws, Satellite Orbits & Energy. Work and Kinetic Energy: Work done by Constant and Variable Forces: Gravitational and Spring, Forces, Power, Conservative and Non-conservative Forces, Work and Potential Energy, Isolated Systems and Conservation of Mechanical Energy, Work Done by External Forces including Friction and Conservation of Energy. System of Particles: Motion of a System of Particles and Extended Rigid Bodies, Center of Mass and Newton's Laws for a System of Particles, Linear Momentum, Impulse, Momentum & Kinetic Energy in One and Two Dimensional Elastic and Inelastic Collisions. Rotational Motion: Rotation about a Fixed Axis, Angular Position, Angular Displacement, Angular Velocity and Angular Acceleration, Rotation under Constant Angular Acceleration, relationship between Linear and Angular Variables, Rotational Inertia, Parallel-axis Theorem, Torque and Newton's Law for Rotation, Work and Rotational Kinetic Energy, Power, Rolling Motion, Angular Momentum for a single Particle and a System of Particles, Conservation of Angular Momentum, Precession of a Gyroscope, Static Equilibrium involving Forces and Torques, Determination of moment of inertia of various shapes i.e. for disc, bar and solid sphere.Angular Momentum: Angular Velocity, Conservation of angular momentum, effects of Torque and its relation with angular momentum. Simple Harmonic Motion (SHM): Amplitude, Phase, Angular Frequency, Velocity and Acceleration in SHM, Linear and Angular Simple Harmonic Oscillators, Energy in SHM, Simple Pendulum, Physical Pendulum, SHM and Uniform Circular Motion, Damped Harmonic Oscillator.

Intended Learning Outcomes:

On successful completion of this course the students will be able to

- give the basic concept of vectors and scalars, divergence and Stokes theorem, work & energy, system of particles, momentum, collision & conservation laws.
- Uses of the above concepts in daily life in a scientific way.
- Intellectual/cognitive skills. Having knowledge of this course, the student will be able to understand vector & and scalar operation**,** dynamics of uniform circular motion, constant, variable & pseudo forces, projectile motion**,** work energy theorem, and system particles.

Recommended Books:

1. D. Halliday, R. Resnick and J. Walker, 9th ed. (2010) "Fundamentals of Physics", John Wiley & Sons.
2. R. A. Serway and J. W. Jewett, 8th ed. (2010), "Physics for Scientists and Engineers", Golden Sunburst Series.
3. R. A. Freedman, H. D. Young, and A. L. Ford (Sears and Zeemansky), Addison-Wesley-Longman, 13th International ed. (2010). "University Physics with Modern Physics".
4. F. J Keller, W. E. Gettys and M. J. Skove, 2th ed. (1992). "Physics: Classical and Modern, McGraw Hill.

5. D. C. Giancoli, 4th ed. (2008). "Physics for Scientists and Engineers, with Modern Physics", Addison-Wesley.

Course Title: Functional English	**Course Code: ENG-303**
Course Structure: Lectures: 3, Labs: 0	**Credit Hours: 3**
Prerequisites: Nil	
Course Objective: This course will familiarize students with the essential language skills for effective communication in diverse real-world scenarios. It focuses on developing proficiency in English language and usage: word choices, grammar and sentence structure. In addition, the course will enable students to grasp subtle messages and tailor their communication effectively through the application of comprehension and analytical skills in listening and reading. Moreover, the course encompasses a range of practical communication aspects including professional writing, public speaking and everyday conversation ensuring that students are equipped for both academic and professional spheres.	
Course Outline: **1.** Vocabulary Building (contextual usage, synonyms, antonyms, and idiomatic expressions) Communicative Grammar (subject-verb agreement, verb tenses, fragments, run-ons, modifiers, articles, word classes etc.) Word Formation (affixation, compounding, clipping, back formation etc.) Sentence Structure (simple, compound, complex and compound-complex). Comprehension and Analysis.**2**. Understanding Purpose, audience and context a. (reading for meaning, descriptive texts versus narrative texts, argumentative texts versus persuasive texts) **3**. Contextual Interpretation (tones, biases, stereotypes, assumptions, inferences etc) **4**. Reading Strategies (skimming, scanning, SQ4R, critical reading) **5**. Active Listening (overcoming listening barriers, focused listening). Effective Communication Principles of Communication (clarity, coherence, correctness and courteousness). Structuring Documents (introduction, body, conclusion and formatting). Inclusivity in Communication (gender-neutral language and cross-cultural communication). Public Speaking (Speech/presentation: extemporaneous and prepared, public announcements and overcoming stage fright) Presentation Skills: a. (the elements of an effective presentation, using visual displays to	

present key facts, figures, charts, and graphs, steps to preparing an effective presentation, one-minute presentations and evaluate presentations, Informal Communication (small talk and networking), Professional Writing (business e-mails, memos, reports, formal letters etc.)
Course Outcomes: By the end of the course the students will be able to apply the enhanced English skills, comprehend a variety of literary and non-literary texts, and express effectively in spoken and written English in diverse social and cultural contexts
Recommended Books: Latest Edition of the Following Books. 1. Murphy, Raymond. Grammar in Use Intermediate Student's Book without Answers. Cambridge University Press, 2018. 2. Kaufman, Lester, and Jane Straus. The Blue Book of Grammar and Punctuation. 2021. 3. Axelrod, Rise B., and Charles R. Cooper. The St. Martin's Guide to Writing [with Access Code]. 2016. 4. Johnson-Sheehan, Richard, and Charles Paine. Writing Today. Pearson, 2019. 5. https://www.hec.gov.pk/english/services/universities/RevisedCurricula/Documents/2011-2012/Education/English2_Sept13.pdf

Course Name: Elements of Set Theory and Mathematical Logic	**Course Code: MTH-312**
Course Structure: Lectures: 3, Lab:0	**Credit Hours: 3**
Prerequisites: Knowledge of Intermediate Mathematics	
<u>Specific Objectives of course:</u> Everything mathematicians do can be reduced to statements about sets, equality and membership which are basics of set theory. • This course introduces these basic concepts. • The course aims at familiarizing the students with cardinals, relations and fundamentals of	

propositional and predicate logics.

Course Outline:

Set theory: Sets, subsets, operations with sets: union, intersection, difference, symmetric difference, Cartesian product and disjoint union. Functions: graph of a function. Composition; injections, surjections, bijections, inverse function. **Computing cardinals:** Cardinality of Cartesian product, union. Cardinality of all functions from a set to another set. Cardinality of all injective, surjective and bijective functions from a set to another set. Infinite sets, finite sets. Countable sets, properties, examples (Z, Q). R is not countable. R, RxR, RxRxR have the same cardinal. Operations with cardinal numbers. Cantor-Bernstein theorem.**Relations:** Equivalence relations, partitions, quotient set; examples, parallelism, similarity of triangles. Order relations, min, max, inf, sup; linear order. Examples: N, Z, R, $P(A)$. Well ordered sets and induction. Inductively ordered sets and Zorn's lemma.**Mathematical logic:**Propositional Calculus. Truth tables. Predicate Calculus.

Learning Outcomes:

Upon completion of the course, the student will be able to

- Students will have a sound knowledge of set theoretic language and be able to use it to codify mathematical objects.
- They will have an appreciation of the notion of infinity and arithmetic of the cardinals and ordinals.
- They will have developed a deep understanding of the Axiom of Choice, Zorn's Lemma and well-ordering principle, and have begun to appreciate the implications.

Recommended Books:

1. Hrbacek, K., & Jech, T. (2017). *Introduction to set theory, revised and expanded*. Crc Press.
2. Hausdorff, F. (2021). *Set theory* (Vol. 119). American Mathematical Soc..
3. Barwise, J. (2017). *Admissible sets and structures* (Vol. 7). Cambridge University Press.
4. Levy, A. (2012). *Basic set theory*. Courier Corporation.
5. Hrbacek, K., & Jech, T. (2017). *Introduction to set theory, revised and expanded*. Crc Press.
6. Kaplansky, I. (2020). *Set theory and metric spaces* (Vol. 298). American Mathematical Society.
7. Cenzer, D., Larson, J., Porter, C., & Zapletal, J. (2020). *Set Theory and Foundations of Mathematics: An Introduction to Mathematical Logic: Volume I: Set Theory*.
8. DeLancey, C. (2017). *A concise introduction to logic*. Open SUNY Textbooks.

SEMESTER-II

Course Name: Calculus-II	**Course Code: MTH-321**
Course Structure: Lectures: 4, Lab:0	**Credit Hours: 04**
Prerequisites: Calculus-I	

Specific Objectives of course:

- This is second course of Calculus.
- As continuation of Calculus I, it focuses on techniques of integration and applications of integrals.
- The course also aims at introducing the students to infinite series, parametric curves and polar coordinates.

Course Outline:

Integration: Anti derivatives and integrals. Riemann sums and the definite integral. Properties of Integral. The fundamental theorem of calculus. The substitution rule. **Techniques of integration**: Integrals of elementary, hyperbolic, trigonometric, logarithmic and exponential functions. Integration by parts, substitution and partial fractions. Approximate integration. Improper integrals. Gamma functions. **Applications of integrals:** Area between curves, average value, Volumes, Arc length. Area of a surface of revolution. Applications to Economics, Physics, Engineering and Biology. **Infinite series**: Sequences and series. Convergence and absolute convergence. Tests for convergence: divergence test, integral test, p-series test, comparison test, limit comparison test, alternating series test, ratio test, root test. Power series. Convergence of power series. Representation of functions as power series. Differentiation and integration of power series. Taylor and Maclaurin series.Approximations by Taylor polynomials.**Conic section, parameterized curves and polar coordinates**: Curves defined by parametric equations. Calculus with parametric curves: tangents, areas, arc length. Polar coordinates. Polar curves, tangents to polar curves. Areas and arc length in polar coordinates.

Learning Outcomes:

Upon completion of the course, the student will be able to

- Interpret a volume of revolution of a function's graph around a given axis as a (Riemann) sum of disks or cylindrical shells, convert to definite integral form and compute its value.
- Express the length of a curve as a (Riemann) sum of of linear segments, convert to definite integral form and compute its value.
- Express the surface area of revolution of a function's graph around a given axis as a (Riemann) sum of rings, convert to definite integral form and compute its value.
- Anti differentiate products of functions by parts.
- Recognize and implement appropriate techniques to anti-differentiate products o trigonometric

functions. 6. devise and apply a trigonometric substitution in integrals involving Pythagorean quotients.

- Interpret the concept of a series as the sum of a sequence, and use the sequence of partial sums to determine convergence of a series.
- Use comparison with a corresponding integral with other series to decide whether infinite series (including p-series) converge or diverge devise parametric representations for conic sections and other relations.
- Compute the length of a curve segment from its parametric representation.
- Apply basic anti differentiation techniques to selected problems arising in various fields such as physical modeling , economics and population dynamics.

Recommended Books:

1. Bressoud, D., Ghedamsi, I., Martinez-Luaces, V., & Törner, G. (2016). *Teaching and learning of calculus*. Springer Nature.
2. Hughes-Hallett, D., Gleason, A. M., & McCallum, W. G. (2020). *Calculus: Single and multivariable*. John Wiley & Sons.
3. Stewart, J., Clegg, D. K., & Watson, S. (2020). *Multivariable calculus*. Cengage Learning.
4. Anton, H., Bivens, I. C., & Davis, S. (2021). *Calculus*. John Wiley & Sons.
5. Bressoud, D., Ghedamsi, I., Martinez-Luaces, V., & Törner, G. (2016). *Teaching and learning of calculus*. Springer Nature.
6. Stewart, J. (2018). *Single variable calculus: Concepts and contexts*. Cengage Learning.
7. Toeplitz, O. (2018). *The calculus: a genetic approach*. University of Chicago Press.

Course Title: Social Science	**Course Code: 000**
Course Structure: Lectures: 3, Labs: 0	**Credit Hours: 3**
Prerequisites: Nil	
Course Objective:	
Course Outline: The course contents will be taken from the booklet of mandatory and general courses.	
Course Outcomes:	
Recommended Books: Latest Edition of the Following Books.	

Course Title: Expository Writing	**Course Code: ENG-304**
Course Structure: Lectures: 3, Labs: 0	**Credit Hours: 3**
Prerequisites: Nil	
Course Objective:	
Course Outline: The course contents will be taken from the booklet of mandatory and general courses.	
Course Outcomes:	
Recommended Books: Latest Edition of the Following Books.	

Course Name: Discrete Mathematics	**Course Code:** MTH-322
Course Structure: Lectures: 3, Lab:0	**Credit Hours:** 3
Prerequisites: Mathematics at intermediate level	

Specific Objectives of course:

Discrete Mathematics is study of distinct, un-related topics of mathematics; it embraces topics from early stages of mathematical development and recent additions to the discipline as well. The present course restricts only to

- counting methods, relations and graphs.
- The objective of the course is to inculcate in the students the skills that are necessary for decision making in non-continuous situations.

Course Outline:

Counting methods: Basic methods: product, inclusion-exclusion formulae. Permutations and combinations. Recurrence relations and their solutions. Generating functions. Double counting. Applications. Pigeonhole principle, applications.**Relations:** Binary relations, n-array Relations. Closures of relations. Composition of relations, inverse relation.**Graphs:** Graph terminology. Representation of graphs. Graphs isomorphism. Algebraic methods: the incidence matrix. Connectivity, Eulerian and Hamiltonian paths. Shortest path problem. Trees and spanning trees.

Complete graphs and bivalent graphs.

Learning Outcomes:

After the successful completion of course, the students will be able to

- Solve basic problems demonstrating the understanding of fundamental concepts of logic, reasoning, algorithms, graphs, counting and grammars.
- Apply the knowledge learnt to solve simple mathematical problems in computer science.

Recommended Books:

1. Toth, C. D., O'Rourke, J., & Goodman, J. E. (Eds.). (2017). *Handbook of discrete and computational geometry*. CRC press.
2. Bannai, E., Bannai, E., Ito, T., & Tanaka, R. (2021). *Algebraic combinatorics* (Vol. 5). Walter de Gruyter GmbH & Co KG.
3. Hein, J. L. (2015). *Discrete structures, logic, and computability*. Jones & Bartlett Learning.
4. Conradie, W., & Goranko, V. (2015). *Logic and discrete mathematics: a concise introduction*. John Wiley & Sons.
5. Biswal, P. C. (2015). *Discrete mathematics and graph theory*. PHI Learning Pvt. Ltd.
6. Rosenstein, J. G. (2020). Discrete mathematics in 21st century education: An opportunity to retreat from the rush to calculus. In *Foundations for the future in mathematics education* (pp. 211-223). Routledge.
7. DeLancey, C. (2017). *A concise introduction to logic*. Open SUNY Textbooks.

Course Title: Natural Science	**Course Code: 000**
Course Structure: Lectures: 3, Labs: 0	**Credit Hours: 3**
Prerequisites: Nil	
Course Objective:	
Course Outline: The course contents will be taken from the booklet of mandatory and general courses.	
Course Outcomes:	
Recommended Books: Latest Edition of the Following Books.	

Course Title: Application of Information and communication technologies (ICT)	**Course Code: CSC-308**
Course Structure: Lectures: 3, Labs: 0	**Credit Hours: 3**
Prerequisites: Nil	
Course Objective:	
Course Outline: The course contents will be taken from the booklet of mandatory and general courses.	
Course Outcomes:	
Recommended Books: Latest Edition of the Following Books.	

Course Title: Introduction to Entrepreneurship	**Course Code: MS-309**
Course Structure: Lectures: 3, Labs: 0	**Credit Hours: 3**
Prerequisites: Nil	
Course Objective: This course is designed to promote entrepreneurial spirit and outlook among students, encouraging them to think critically, identify opportunities, and transform their ideas into successful ventures. It aims at imparting them with the requisite knowledge, skills, abilities, enabling them to seize the identified opportunities for initiating ventures and successfully navigating the challenges that come with starting a business and managing it. The course covers topics relevant to entrepreneurship including setting up and initiation of business, market research, opportunity identification, business Lanning, financial literacy for managing finances and securing funding, marketing and sales, team building and innovation.	
Course Outline: Introduction to Entrepreneurship, Entrepreneurial Skills, . Opportunity Recognition and Idea Generation, Opportunity identification, evaluation and exploitation, Innovative idea generation techniques for entrepreneurial ventures, Marketing and Sales, Financial Literacy, Team Building for Startups & Regulatory Requirements to Establish Enterprises in Pakista**n**	
Course Outcomes: Upon successful completion of the course participants will: ● Have a basic understanding of the Islamic World and Muslim beliefs. ● Know the origins of the Islamic Banking and Finance. ● Appreciate the rationale behind the development of the Islamic finance industry.	

- Be able to assess the nature and scope of the Islamic finance industry in relation to its conventional counterpart.
- ·Develop an appropriate level of understanding of the main principles of Islamic banking and finance.
- ·Acquire essential knowledge about the key Islamic financial contracts, as used by the industry.
- ·Know about Murabaha and Musharaka contracts, Ijara and Istisna'a financing methods,as well as Salam and Takaful insurance.
- Be familiarized with the Islamic financial infrastructure, international financial institutions, and regulatory bodies.

Recommended Books: Latest Edition of the Following Books.

1. Barringer, B. R., & Ireland, R. D. (2012). Entrepreneurship: Successfully Launching New Ventures. Pearson.
2. Kuratko, Donald F. (2017). Entrepreneurship : Theory, Process, Practice (ed.10). United State of America: Cengage Learning. Timmons, J. A., & Spinelli, S. (2003). New venture creation/entrepreneurship for the 21st century. Singapore City: McGraw-Hill.
3. Abrams, R. (2017). Entrepreneurship: A Real-World Approach (2nd ed., illustrated). Planning Shop.
4. Read, S., Sarasvathy, S., Dew, N., & Wiltbank, R. (2016). Effectual Entrepreneurship (2nd ed.). Routledge. https://doi.org/10.4324/9781315684826
5. Ries, E. (2011).The Lean Startup: How Today's Entrepreneurs Use Continuous Innovation to Create Radically Successful Businesses.

SEMESTER-III

Course Name: **Calculus-III**	**Course Code:** MTH-411
Course Structure: Lectures: 4, Lab:0	**Credit Hours: 04**
Prerequisites: Calculus-II	

Specific Objectives of course:

This is third course of Calculus and builds up on the concepts learned in first two courses. The students would be introduced

- to the vector calculus
- the calculus of multivariable functions
- double and triple integrals along with their applications.

Course Outline:

Vectors and analytic geometry in space: Coordinate system. Rectangular, cylindrical and spherical coordinates. The dot product, the cross product. Equations of lines and planes. Quadric surfaces.**Vector-valued functions**: Vector-valued functions and space curves. Derivatives and integrals of vector valued functions. Arc length. Curvature, normal and binormal vectors.**Multivariable functions and partial derivatives**: Functions of several variables. Limits and Continuity. Partial derivatives, Composition and chain rule. Directional derivatives and the gradient vector. Implicit function theorem for several variables. Maximum and minimum values. Optimization problems. Lagrange Multipliers.**Multiple integrals**: Double integrals over rectangular domains and iterated integrals. Non-rectangular domains. Double integrals in polar coordinates. Triple integrals in rectangular, cylindrical and spherical coordinates. Applications of double and triple integrals. Change of variables in multiple integrals.**Vector calculus**: Vector fields. Line integrals. Green's theorem. Curl and divergence. Surface integrals over scalar and vector fields. Divergence theorem. Stokes' theorem

Learning Outcomes:

After the successful completion of course, the students will be able to

- Write vectors in component form and as linear combinations of standard unit vectors, Add and subtract vectors algebraically and graphically. Find the angle between two vectors and the direction cosines of a vector in space. Find limits, derivatives, and integrals of vector-valued functions.
- Determine if a function of several variables is continuous or differentiable. partial derivatives of a function of several variables and use the chain rules, approximation techniques to functions of several variables using differentials.
- Find the directional derivative and gradient of a function of several variables, equation of the tangent plane and normal line to a surface at a point and Solve optimization problems involving functions of several variables.
- Evaluate double and triple integrals as iterated integrals. Find the area or volume using double or triple integrals. Determine if a vector field is conservative and find a potential function, divergence and curl of a vector field.
- Find a set of parametric equations of a surface in space. Apply the Divergence Theorem and Stokes' Theorem to evaluate line integrals or flux integrals.

Recommended Books:

1. Stewart, J., Clegg, D. K., & Watson, S. (2020). *Multivariable calculus*. Cengage Learning.
2. Lax, P. D., & Terrell, M. S. (2017). *Multivariable calculus with applications*. Cham, Switzerland: Springer.

3. Taylor, M. E. (2020). *Introduction to Analysis in Several Variables: Advanced Calculus* (Vol. 46). American Mathematical Soc.
4. Hughes-Hallett, D., Gleason, A. M., & McCallum, W. G. (2020). *Calculus: Multivariable*. John Wiley & Sons.
5. Stewart, J., Clegg, D. K., & Watson, S. (2020). *Multivariable calculus*. Cengage Learning.
6. Walschap, G. (2015). *Multivariable Calculus and Differential Geometry*. Walter de Gruyter GmbH & Co KG.

Quantitative Reasoning-I	**Course Code: MTH-401**
Course Structure: Lectures:3	**Credit Hours: 3**
Prerequisites:	

Specific Objectives of Course

- Quantitative reasoning (I) as in introductory-level undergraduate course that focuses on the fundamentals related to the quantitative concept and analysis.
- The course is design to familiarize students with the basic concepts of mathematics and statistics and to develop students' ability to analyze and interpret quantitative information. Through a combination of theoretical concepts and practical exercises
- This course will also enable students cultivate their quantitative literacy and problem-solving skills while effectively expending their academic horizon and breadth of knowledge of their specific major/field of study

Course Outline:

Numerical Literacy Number system and basic arithmetic operation;Units and their conversion, dimension, area, parameter, and volume;Rates, ratio, proportion, and percentage;Types and sources of data;Measurement scales;Table and graphical presentation of data;Quantitative reasoning exercises using number knowledge. **Fundamental Mathematical Concept**Basic of geometry (lines, angles, circles, polygons etc); Sets and their operations; Relations, functions, and their graphs; Exponent, factoring and simplifying algebraic expression;Algebraic and graphical solutions of linear and quadratic equations and inequalities; Quantitative reasoning exercises using fundamental mathematical concepts; **Fundamental Statistical Concepts** Population and sample; Measure of central tendency, dispersion and data interpretation; Rules of counting (multiplicative, permutation, and combination); Basic probability theory; Introduction to random variables and their probability distribution;Quantitative reasoning exercises using fundamental statistical concept;

Learning Outcomes

- By the end of this course, student shall have:
- Fundamental numerical litracy to enable them work with munbers understand their meaning and present data accurately;

- Understanding of fundamental mathematical and statistical concept;
- Basic ability to interpret data presented and various format including but not limited to tables, graphs, charts, and equations etc.

Recommended Books:

1. "Quantitative Reasoning: Tools for Todays Informs Citizen" by Bernard L. Madison. Lynn and Arthur Steen
2. "Quantitative Reasoning for the Information Age" by Bernard L. Madison. And David M. Bressoud.
3. "Fundamental of Mathematics" by Wade Ellis.
4. "Quantitative Reasoning: Thinking and Numbers" by Eric Zaslow.
5. "Thinking Clearly with Data: A Guide to Quantitative Reasoning and Analysis" by Ethan Bueno De Mesquita and Anthony Fowler.
6. "Using and Understanding Mathematics: A Quantitative Reasoning Approach" by Bennett, J. O., Briggs, W. L., & Badalamenti, A.
7. "Decreet Mathematics and Its Application" by Kenneth H. Rosen.
8. "Statistics for Technologies: A Course in Applied Statistics" by Chatfield, C.
9. "Statistics: Unlocking the Power of Data" by Robin H. Lock, Patti Farzer Lock, Kari Lock, Morgan and Eric F. Lock.

Course Name: **Group Theory**	**Course Code:** MTH-412
Course Structure: Lectures: 3, Lab:0	**Credit Hours: 03**
Prerequisites: Elements of Set Theory and Mathematical Logic	

Specific Objectives of course:

- This course introduces basic concepts of groups and their homomorphisms.
- The main objective of this course is to prepare students for courses which require a good back ground in group theory like Rings and Modules, Linear Algebra, Group Representation, Galois Theory etc.

Course Outline:

Groups: Definition of a group, subgroup, subgroup generated by a set. The cyclic groups, cosets and Lagrange's theorem. Normalizer centralizer. The center of a group. Equivalence relation in a group, conjugacy classes. Normal subgroups, quotient group. **Group homomorphisms:** Homomorphisms and isomorphism and Automorphism. Kernel and image of homomorphism. Isomorphism theorems. Permutation groups. The cyclic decomposition of a permutation group. Cayley's theorem. Direct product of two groups and examples.

Learning Outcomes:

At the end of the course the student should be able to demonstrate knowledge of the structure of Groups.

- Be able to state the group axioms and to verify whether a given set and binary operation form a group.

- Define subgroup, identity element, inverse, associativity, order of an element, order of a group, group table, inverse, cyclic group, abelian/commutative group,
- Compute orders, powers, and inverses in concrete examples.
- Determine whether a group is cyclic or abelian.
- Determine whether a given subset is a subgroup, including using Lagrange's theorem.
- Be able to define and compute with cyclic groups, the additive group mod n, the multiplicative group mod p, the symmetric group, the dihedral group

Recommended Books:

1. Shapira, Y. (2023). Linear Algebra and Group Theory for Physicists and Engineers. Germany: Springer International Publishing.
2. Davvaz, B. (2021). A First Course in Group Theory. Singapore: Springer Nature Singapore.
3. Dixon, M. R., Kurdachenko, L. A., Subbotin, I. Y. (2017). Ranks of Groups: The Tools, Characteristics, and Restrictions. United Kingdom: Wiley.
4. Druțu, C., Kapovich, M. (2018). Geometric Group Theory. United States: American Mathematical Society.
5. Interactions Between Group Theory, Symmetry and Cryptology. (2020). Switzerland: Mdpi AG.
6. Roman, S. (2011). Fundamentals of Group Theory: An Advanced Approach. United Kingdom: Birkhäuser Boston.

Course Title: Civic & Community Engagement	**Course Code: PSC-418**
Course Structure: Lectures: 3, Labs: 0	**Credit Hours: 3**
Prerequisites: Nil	
Course Objective:	
Course Outline: The course contents will be taken from the booklet of mandatory and general courses.	
Course Outcomes:	
Recommended Books: Latest Edition of the Following Books.	

SEMESTER-IV	
Course Name: **Computing Tools**	**Course Code:** MTH-421
Course Structure: Lectures: 3, Lab:0	**Credit Hours: 03**
Prerequisites: Elements of Set Theory and Mathematical Logic	

Specific Objectives of course:

This course emphasizes

- program structure as well as functional and rule-based programming which is compared to more traditional programming,
- to help students understand and use Mathematica's unique features to their advantage.
- In the course students will learn how to solve particular problems more efficiently by choosing the appropriate programming paradigm.

Course Outline:

Introduction General introduction and basic use of mathematica, numeric and symbolic computation, the note book, working with data, input and output, built-in functions, front end and the kernel, errors, help **Language of Mathematica** Expressions, values, variables, functions and assignments,, immediate vs delayed, patterns and pattern matching, conditional patterns, predicates and Boolean operations, relational and logical operators, attributes. **Lists** Simple and multidirectional list, List construction and manipulation, testing a list, extracting elements, rearranging list, list component assignments, working with several lists **Programming** Functional programming, Map, Thread, Apply, Inner and Outer, Nest, NestList,Programs as functions, user defined functions, pure functions, and module. Procedural programming, loops, flow control. Rule base programming. Dynamic programming. Graphics programming. Writing packages.

Learning Outcomes:

After finishing this course, students will able to

- Know about programming language
- Will solve algebraic calculations through software
- Plot graphs in 2D, 3D, Contour plot
- Also solve linear and non-linear differential equations with initial and boundary conditions
- Solve matrices

Recommended Books:

1) Smith, E. (2022). Introduction to the Tools of Scientific Computing. Poland: Springer International Publishing.
2) Applied Reconfigurable Computing. Architectures, Tools, and Applications: 18th International Symposium, ARC 2022, Virtual Event, September 19–20, 2022, Proceedings. (2022). Poland: Springer Nature Switzerland.
3) Past, Present and Future of Computing Education Research: A Global Perspective. (2023). Germany: Springer International Publishing.
4) Cloud Computing Tools And Techniques. (2023). (n.p.): AG PUBLISHING HOUSE (AGPH Books).
5) Peters, B., Peters, T. (2018). Computing the Environment: Digital Design Tools for Simulation and Visualisation of Sustainable Architecture. United Kingdom: Wiley.
6) Computing Tools for Modeling, Optimization and Simulation: Interfaces in Computer Science and Operations Research. (2012). United States: Springer US.

Course Name: Linear Algebra	**Course Code:** MTH-422
Course Structure: Lectures: 3, Lab:0	**Credit Hours:** 3
Prerequisites: Calculus-I	

Specific Objectives of course:

Linear algebra is the study of vector spaces and linear transformations. The main objective of this course is to

- help students learn in rigorous manner, the tools and methods essential for studying the solution spaces of problems in mathematics, engineering, the natural sciences, and social sciences
- to develop mathematical skills needed to apply these to the problems arising within their field of study; and to various real-world problems.

Course Outline:

System of Linear Equations: Representation in matrix form. Matrices. Operations on matrices. Echelon and reduced echelon form. Inverse of a matrix (by elementary row operations). Solution of linear system. Gauss-Jordan method. Gaussian elimination. **Determinants:** Permutations of order two and three and definitions of determinants of the same order. Computing of determinants. Definition of higher order determinants. Properties. Expansion of determinants.**Vector Spaces:** Definition and examples, subspaces. Linear combination and spanning set. Linearly Independent sets. Finitely generated vector spaces. Bases and dimension of a vector space. Operations on subspaces, Intersections, sums and direct sums of subspaces. Quotient Spaces. **Linear mappings:** Definition and examples. Kernel and image of a linear mapping. Rank and nullity. Reflections, projections, and homotheties. Change of basis. Eigen-values and eigenvectors. Theorem of Hamilton-Cayley. **Inner product Spaces:** Definition and examples. Properties, Projection. Cauchy inequality. Orthogonal and orthonormal basis. Gram Schmidt Process. Diagonalization.

Learning Outcomes:

After completing this course, students will able to

- Solve systems of linear equations using multiple methods, including Gaussian elimination and matrix inversion.
- Carry out matrix operations, including inverses and determinants.
- Demonstrate understanding of the concepts of vector space and subspace.
- Demonstrate understanding of linear independence, span, and basis.
- Determine eigenvalues and eigenvectors and solve eigenvalue problems.
- Apply principles of matrix algebra to linear transformations.
- Demonstrate understandingof inner products and associated norms.

Recommended Books:

1) Messer, R. (2021). Linear Algebra: Gateway to Mathematics. United States: American Mathematical Society.
2) Anton, H., Kaul, A. (2019). Elementary Linear Algebra. United Kingdom: Wiley.
3) Matrices and Linear Algebra With GAP. (2020). (n.p.): Akhilesh Chandra Yadav.
4) Neri, F. (2019). Linear Algebra for Computational Sciences and Engineering. Germany: Springer International Publishing.
5) Johnston, N. (2021). Introduction to Linear and Matrix Algebra. India: Springer International Publishing.
6) O'Leary, M. L. (2021). Linear Algebra. United Kingdom: Wiley.

Course Title: Probability Theory	**Course Code: STAT-403**
Course Structure: Lectures: 3, Labs: 0	**Credit Hours: 3**
Prerequisites: NIL	

Course Objective:
The course is designed to introduce the fundamentals of probability theory and its applications. To provide knowledge of basic laws of probability, random variables, random processes and probability distributions

Course Outline:
Introduction to Probability theory; counting techniques; Permutation, Combination. Random experiment, event, sample space (continuous and discrete). Laws of probability, conditional

probability, independent events. Bayes theorem. Random variables; Mean and variance of a discrete random variable. Probability Distributions; Discrete Probability Distribution and its properties. Bernoulli trials, Binomial and Poisson distributions. Continuous Random Variable, probability density function and its properties. Uniform distribution, Normal Distribution and its properties. Expectation; Moments, Expectation of a function of a random variable, characteristic function.

Course Outcomes: After completing the course, the students will be able to

- Demonstrate the knowledge of probability and probability distributions.
- Apply basic counting techniques (multiplication rule, combinations, and permutations) to compute probability and odds.

Recommended Books: Latest Edition of the Following Books.

1. Clark, G.M. and Cooke, D. (1998), "A Basic Course in Statistics" 4th ed, Arnold, London.
2. Chaudhry. S.M.and Kamal, S. (1996), "Introduction to Statistical Theory" Parts I & II, 6th ed, Ilmi Kitab Khana, Lahore, Pakistan.
3. Mclave, J.T., Benson, P.G. and Snitch, T. (2005) "Statistics for Business & Economics" 9t ed, Prentice Hall, New Jersey.
4. Spiegel, M.R., Schiller, J.L. and Sirinivasan, R.L. (2000) "Probability and Statistics", 2nd ed. Schaums Outlines Series. McGraw Hill. NY.
5. Walpole, RE., Myers, R.H and Myers, S.L. (1998), 'Probability and Statistics for Engineers and Scientist" 6th edition, Prentice Hall, NY.
6. Weiss, N.A. (1997), "Introductory Statistics" 4th ed. Addison-Wesley Pub. Company, Inc.

Course Name: Number Theory	**Course Code:**MTH-423
Course Structure: Lectures: 3, Lab:0	**Credit Hours:** 3
Prerequisites: Linear Algebra	

Specific Objectives of course:

The focus of the course is on study

- of the fundamental properties of integers
- develops ability to prove basic theorems
- Also include study of division algorithm, prime numbers and their distributions
- Diophantine equations, and the theory of congruences.

Course Outline:

Preliminaries: Well-ordering principle. Principle of finite induction. **Divisibility theory:** The division algorithms. Basis representation theorem. Prime and composite numbers. Canonical decomposition. The greatest common divisor. The Euclidean algorithm. The fundamental theorem of arithmetic. Least common multiple. **Linear Diophantine equations:** Congruences. Linear congruences. System of linear congruences. The Chinese remainder theorem. Divisibility tests. Solving polynomial congruences. Fermat's and Euler's theorems. Wilson's theorem. **Arithmetic functions:** Euler's phi-function. The functions of J and sigma. The Mobius function. The sieve of Eratosthenes. Perfect numbers. Fermat and Mersenne primes. **Primitive Roots and Indices:** The order of integer mod n. Primitive roots for primes. Composite numbers having primitive roots. **Quadratic residues:** Legendre symbols and its properties. The quadratic reciprocity law. Quadratic congruences with composite moduli. Pythagorean triples. Representing numbers as sum of two squares.

Learning Outcomes:

After studying this course, you should be able to:

- find quotients and remainders from integer division
- apply Euclid's algorithm and backwards substitution
- understand the definitions of congruences, residue classes and least residues
- add and subtract integers, modulo n, multiply integers and calculate powers, modulo n
- determine multiplicative inverses, modulo n and use to solve linear congruences.

Recommended Books:

1. Nagell, T. (2021). Introduction to Number Theory. United States: American Mathematical Society.
2. Number Theory and Its Applications. (2020). United Kingdom: IntechOpen.
3. Stewart, I., Tall, D. O. (2020). Algebraic Number Theory and Fermat's Last Theorem. United Kingdom: CRC Press, Taylor et Francis Group.
4. Wilson, R. J. (2020). Number Theory: A Very Short Introduction. United Kingdom: Oxford University Press.
5. Grigorieva, E. (2018). Methods of Solving Number Theory Problems. Germany: Springer International Publishing.
6. Fine, B., Rosenberger, G. (2018). Number Theory: An Introduction Via the Density of Primes. Switzerland: Springer International Publishing.

Course title: Quantitative Reasoning II	Course Code: MTH-402
Course Structure: Lectures:3	Credit Hours: 3

Prerequisites:

Specific Objectives of Course

Quantitative reasoning (II) is a sequential undergraduate course that focuses on logical reasoning supported with mathematical and statistical concepts and modelling/analysis technique to equip student with analytical skills and crtitical thinking abilities necessary to navigate the complexities of the modern world. The course is design to familiarize students with the quantitative concept and technique require to interpret and analyze numerical data to inculcate and ability in students the logical reasoning to construct and evaluate arguments, identify fallacies, and think systematically. Keeping the prerequisite course of quantitative reasoning I and its base, this course will enable students further their quantitative, logical and critical reasoning abilities to complement their specific major/field of study.

Course Outline:

Logic and Logical Critical Reasoning Introduction and Importance of logic; Inductive, deductive, and abductive approaches of reasoning; Prepositions, arguments (valid; invalid), logical connectives, truth tables and prepositional equivalences; Logical fallacies; Venn diagram; Predicates and quantifiers Quantitative reasoning exercises using logical reasoning concepts and techniques; **Mathematical Modelling and Analysis** Introduction to deterministic models Use of linear functions for modelling in real world situations; Modelling with system of linear equation and their solutions; Elementary introduction to derivatives and mathematical modelling; Linear and exponential growth and decay models; Quantitative reasoning exercises using mathematical modelling; **Statistical Modelling and Analysis.** Introduction to probabilistic models. Bivariate analysis, scatter plots; Simple linear regression model and correlation analysis; Basics of estimation and confidence interval; Testing of hypothesis (Z-test; T-test); Statistical inference in decision making; Quantitative reasoning exercises and using statistical modelling;

Learning Outcomes

By the end of this course, student shall have:

- Understanding of logic and logical reasoning;
- Understanding of basics quantitative modelling and analysis;
- Logical reasoning skills and abilities to apply them to solve quantitative problems and evaluate arguments;
- Ability to critically evaluate quantitative information to make evidence based decisions through appropriate computational tools:

Recommended Books:

1. "Using and Understanding: A Quantitative Reasoning Approach" by Bennett, J.O., Biggs, W. L., and Badalamenti, A.

2."Discrete Mathematics and Its Applications" by Kenneth H. Rosen.

3."Discrete Mathematics with Applications" by Susanna S. Epp.

4."Applied Mathematics for Business, Economics, and Social Sciences" by Frank S. Budnick.

5.“Elementary Statistics: A Step by Step Approach” by Allan Bluman.

6.“Introductory Statistics” by Prem S. Mann.

7.“Applied Statistical Modelling” by Salvatore Babones.

8.“Barrons SAT” by Sharvonweiner Green, M. A and Lra K. Wolf.

SEMESTER-V

Course Name: Complex Analysis	**Course Code:**MTH-512
Course Structure: Lectures: 3, Lab:0	**Credit Hours:** 3
Prerequisites: Real Analysis-I	

Specific Objectives of course:

This is an introductory course in complex analysis

- To introduce the theories for functions of a complex variable.
- To explore algebraic, geometric and topological structures of the complex number field.
- To introduce the concepts of analyticity, Cauchy-Riemann relations and harmonic functions.
- To present Complex integration and complex power series.
- To discuss the classification of isolated singularities and examine the theory and illustrate the applications of the calculus of residues in the evaluation of integrals.

Course Outline:

Fundamental complex of complex numbers, limit points and closure of a set interior, exterior and boundary of a set. Algebra of complex numbers, analytic functions, C-R equations, harmonic functions, elementary functions, branches of log z, complex exponents. Integrals: Contours, Cauchy-Goursat theorem, Cauchy integral formula, Morera’s theorem, maximum moduli of functions, Liouville’s theorem, fundamental theorem of algebra. Series: Convergence of sequences and series, Taylor series, Laurent series, uniqueness of representation, zeros of analytic function. Residues and poles: the residue theorem, evaluation of improper integrals, integrals involving trigonometric functions, integration around a branch point. Mapping by elementary functions: linear functions, the function 1/z, the transformations w = exp(z) and w = sin(z), successive transformations. Analytic continuation, the argument principle, Rouche’s theorem.

Learning Outcomes:

- Student will have introduced to the fundamental concepts of complex analysis and their role in modern mathematics and applied contexts.
- Student will demonstrate to accurate and efficient use of complex analysis techniques.

- Student will be able to understand capacity for mathematical reasoning through analyzing, proving and explaining concepts from complex analysis
- Student will be able to apply problem-solving using complex analysis techniques applied to diverse situations in physics, engineering and other mathematical contexts.
- Student will be able to apply problem-solving using evaluation of improper integral by Cauchy Residue Theorem.

Recommended Books:

1. Stewart, I., & Tall, D. (2018). *Complex analysis*. Cambridge University Press.
2. Needham, T. (2023). *Visual complex analysis*. Oxford University Press.
3. Asmar, N. H., & Grafakos, L. (2018). *Complex analysis with applications*. Berlin: Springer.
4. Bulboacă, T., Joshi, S. B., & Goswami, P. (2019). *Complex Analysis: Theory and Applications*. Walter de Gruyter GmbH & Co KG.
5. Gonzalez, M. (2018). *Complex analysis: selected topics*. Routledge.

Course Name: Topology	**Course Code:**MTH-511
Course Structure: Lectures: 3, Lab:0	**Credit Hours:** 3
Prerequisites: Calculus-I	

Specific Objectives of course:

- To introduce the student to elementary properties of topological spaces and structures defined on them
- To introduce the student to maps between topological spaces
- To develop the student's ability to handle abstract ideas of Mathematics and Mathematical proofs.

Course Outline:

Motivation and introduction, sets and their operations, countable and uncountable sets, cardinal and transfinite numbers. Topological spaces, open and closed sets, interior, closure and boundary of a set, neighborhoods and neighborhood systems, isolated points, some topological theorems, topology in terms of closed sets, limit points, the derived and perfect sets, dense sets and separable spaces, topological bases, criteria for topological bases, local bases, first and second countable spaces, relationship between sparability and second countablity, relative or induced topologies, necessary and sufficient condition for a subset of a subspace to be open in the original space, induced bases.Metric spaces, topology induced by a metric, equivalent topologies, formulation with closed sets, Cauchy sequence, complete metric spaces, characterization of completeness, Cantor's intersection theorem, the completion of metric space, metrizable spaces. Continuous functions, various characterizations of continuos functions, geometric meaning, homeomorphisms, open and closed continuous functions, topological properties and homeomorphisms. Separation axioms, T1 and T2 spaces and their characterization, regular and normal spaces and their characterizations, Urysohn's lemma, Urysohn'n metrizablity theorem (without proof). Compact spaces their characterization and some theorems, construction of compact spaces, compactness in metric spaces, compactness and completeness, local compactness. Connected spaces, characterization and some properties of connected spaces.

Learning Outcomes:

- Understanding elementary properties of topological spaces and structures defined on them
- Construct maps between topological spaces
- Ability to handle abstract ideas of Mathematics and Mathematical proof
- Demonstrate an understanding of the concepts of metric spaces and topological spaces, and their role in mathematics.
- Demonstrate familiarity with a range of examples of these structures.
- Prove basic results about completeness, compactness, connectedness and convergence within these structures.

Recommended Books:

1. Richmond, T. (2020). General Topology: An Introduction. Germany: De Gruyter.
2. Adhikari, A., Adhikari, M. R. (2022). Basic Topology 1: Metric Spaces and General Topology. South Korea: Springer Nature Singapore.
3. Adhikari, A., Adhikari, M. R. (2022). Basic Topology 2: Topological Groups, Topology of Manifolds and Lie Groups. South Korea: Springer Nature Singapore.
4. Singh, T. B. (2019). Introduction to Topology. Singapore: Springer Nature Singapore.
5. Yan, M. (2016). Introduction to Topology. Germany: De Gruyter.

Course Name: Real Analysis-I	**Course Code:** MTH-514
Course Structure: Lectures: 3, Lab:0	**Credit Hours:** 3
Prerequisites: Calculus-III	

Specific Objectives of course:

- Provide the background information in Analysis necessary for the student to obtain a Bachelor degree in Mathematics or Mathematics Education, or a closely related field.
- Provide prerequisite training for other advanced courses in Real Analysis

Course Outline:

Number Systems: Ordered fields. Rational, real and complex numbers. Archimedean property, supremum, infimum and completeness. **Topology of real numbers**: Convergence, completeness, completion of real numbers. Open sets, closed sets, compact sets. Heine Borel Theorem. Connected sets. **Sequences and Series of Real Numbers:** Limits of sequences, algebra of limits. Bolzano Weierstrass Theorem. Cauchy sequences, liminf, limsup. Limits of series, convergences tests, absolute and conditional convergence. Power series. **Continuity:** Functions, continuity and compactness, existence of minimizers and maximizers, uniform continuity. Continuity and connectedness, Intermediate mean Value Theorem. Monotone functions and discontinuities. **Differentiation**: Mean Value Theorem, L'Hopital's Rule, Taylor's Theorem.

Learning Outcomes:

- Students will be able to demonstrate competence with the algebraic and order properties of real numbers.
- Define and recognize the basic topological properties of R.
- Students will be able to demonstrate competence with properties of real numbers by finding supremum and infimum of sets and using the completeness property of real numbers.
- Students will be able to demonstrate competence with elementary properties of sequences by finding limits and proving results involving sum/difference/product/quotients of sequences.
- Students will be able to apply the monotone convergence theorem to prove convergence of bounded monotone sequences.
- Students will be able to demonstrate ability to use Taylor Theorem, the Mean value Theorem, Cauchy Mean value Theorem and use L'Hôpital's Rule to compute limits of functions.
- Students will be able to define and recognize the continuity of real functions
- Students will be able to define and recognize the differentiability of real functions and its related theorems.

Recommended Books:

1. Ziemer, W. P., & Torres, M. (2017). *Modern real analysis*. Springer.
2. Cummings, J. (2019). *Real analysis: a long-form mathematics textbook*. Create Space Independent Publishing Platform.
3. Silva, C. E. (2019). *Invitation to Real Analysis.* United States: American Mathematical Society.
4. Axler, S. (2020). *Measure, integration & real analysis*. Springer Nature.
5. Laczkovich, M., Sós, V. T. (2016). *Real Analysis: Foundations and Functions of One Variable*. United States: Springer New York.

Course Name: Ordinary Differential Equation	**Course Code:**MTH-513
Course Structure: Lectures: 3, Lab:0	**Credit Hours:** 3
Pre-requisite: calculus & Linear Algebra	

Specific Objectives of course:

- To introduce students to the formulation, classification of differential equations
- existence and uniqueness of solutions.
- To provide skill in solving initial value and boundary value problems.
- To develop understanding and skill in solving first and second order linear homogeneous and non-homogeneous differential equations
- solving differential equations using power series methods.

Course Outline:

Definitions and occurrence of differential equations (d.e.), remarks on existence and uniqueness of solution. First order and simple higher order d.e; special equations of 1st order. Elementary applications of 1st order d.e. Theory of linear differential equations. Linear equations with constant coefficients. Methods of undetermined coefficients and variation of parameters. S-L boundary value problems; self adjoint operators. Fourier series. Series solution of d.c. The Bessel modified Bessel Legendres, Hermite, Hypergeometric, Lauguere equations and their solutions. Orthogonal polynomials. Green function for ordinary differential equations.

Learning Outcomes:

- Student will be able to solve first order differential equations utilizing the standard techniques for separable, exact, linear, homogeneous, or Bernoulli cases.
- Student will be able to find the complete solution of a nonhomogeneous differential equation as a linear combination of the complementary function and a particular solution.
- Student will be introduced to the complete solution of a nonhomogeneous differential equation with constant coefficients by the method of undetermined coefficients.
- Student will be able to find the complete solution of a differential equation with constant coefficients by variation of parameters.
- Student will have a working knowledge of basic application problems described by second order linear differential equations with constant coefficients.

Recommended Books:

1. Hsu, S. B., & Chen, K. C. (2022). *Ordinary differential equations with applications*. World scientific.
2. Sanchez, D. A. (2019). *Ordinary Differential Equations and Stability Theory*: An

Introduction. United States: Dover Publications.
3. David, G., SCHAEFFER, C., & John, W. (2018). *Ordinary differential equations: Basics and beyond*. Springer.
4. Trefethen, L. N., Birkisson, Á., & Driscoll, T. A. (2017). *Exploring ODEs*. Society for Industrial and Applied Mathematics.
5. Han, X., & Kloeden, P. E. (2017). *Random ordinary differential equations and their numerical solution*. Springer Singapore.

Course Name: Numerical Analysis	**Course Code:**MTH-515
Course Structure: Lectures: 3, Lab:0	**Credit Hours:** 3
Prerequisites: Calculus-I, Linear Algebra	

Specific Objectives of course:

This course is an introduction to a broad range of numerical methods for solving mathematical problems that arise in Science and Engineering. The goal is to provide a basic understanding of the

- derivation, analysis, and use of these numerical methods, along with a rudimentary understanding of finite precision arithmetic
- the conditioning and stability of the various problems and methods. This will help you choose, develop and apply the appropriate numerical techniques for your problem, interpret the results, and assess accuracy.

Course Outline:

Operators: Introduction to Error, Forward operator, Backward operator, Shift operator, Average operator, Relations between operators, Difference tables, Factorial Polynomials. **Interpolation (Equally Spaced):** Newton forward formula, Stirling's formula,Newton Backward formula, Guass's interpolation formula, **Interpolation (Unequally Spaced):** Newton divided difference table, Newton formula, Lagrange Interpolating formula, Cubic splines, and Inverse interpolation. **Numerical Differentiation:** Differentiation formulas based on Newton, Stirling's formulas, Differentiation based on Newton divided formulas, Higher order differentiation formulas, Richardson's Extrapolation formulas, and Error analysis of higher order formulas. **Numerical Integration:**Newton Quadrature formulas, Trapezoidal rule, Simpson's rule, Error analysis, Romberg Integration. **Numerical solution of Linear Equations:** Introduction to Direct and Iterative methods, Jacobi method, Guass seidel method, SOR method, Rate of convergence. **Numerical solution of Nonlinear Equations:** Bisection method, Newton raphson method, Secant method, Regula Falsi method, Rate of Convergence and order of convergence.**Numerical Solution to ODE's (Initial Value)**Single Step Methods, Euler method, Taylor's methos, R-K 2 method, Heun's Method, R-K 4 method, Truncation Errors. Muti- Step Methods, Adam Bashforth method, Adam Moulton method, Truncation Errors**. Numerical Solution to ODE's (Initial Boundary Value)**Ray Leigh Ritz method, Finite difference method.**Eigen Values:** Numerically finding Eigen values, Properties and applications, Power method, Jacobi method,

Learning Outcomes:

- Understand the theoretical and practical aspects of the use of numerical analysis.
- proficient in implementing numerical methods for a variety of multidisciplinary applications.
- establish the limitations, advantages, and disadvantages of numerical analysis.
- derive numerical methods for various mathematical operations and tasks, such as interpolation, differentiation, integration, the solution of linear and nonlinear equations, and the solution of differential equations.
- understand of common numerical analysis and how they are used to obtain approximate solutions to otherwise intractable mathematical problems.

Recommended Books:

1. Sutton, B. (2019). *Numerical Analysis: Theory and Experiments*. Society for Industrial and Applied Mathematics.
2. Kharab, A., Guenther, R. (2023). *An Introduction to Numerical Methods: A MATLAB® Approach*. United States: CRC Press.
3. Loustau, J. (2017). *Elements of Numerical Analysis With Mathematica.* Singapore: World Scientific Publishing Company.
4. Novak, K. (2022). *Numerical Methods for Scientific Computing: The Definitive Manual for Math Geeks*. Equal Share Press.
5. Salgado, A. J., & Wise, S. M. (2022). *Classical Numerical Analysis: A Comprehensive Course*. Cambridge University Press.

SEMESTER-VI

Course Name: Partial Differential Equations	**Course Code:**MTH-521
Course Structure: Lectures: 3, Lab:0	**Credit Hours:** 3
Prerequisites: Ordinary Differential Equations	

Specific Objectives of course:

- Introduce students to partial differential equations.
- Introduce students to how to solve linear Partial Differential with different methods.
- Find the solutions of PDEs are determined by conditions at the boundary of the spatial domain and initial conditions at time zero.
- Technique of separation of variables to solve PDEs and analyze the behavior of solutions in terms of eigen function expansions.

Course Outline:

Partial differential equations (p.d.e) of the first order. Nonlinear p.d.e. of first order Applications of 1st order partial differential equations. Partial differential equations of second order: Mathematical modeling of heat, Laplace and wave equations. Classification of 2nd order p.d.e. Boundary and initial conditions. Reduction to canonical form and the solution of 2nd order p.d.e. Technique of separation of variable for the solution of p.d.e with special emphasis on Heat, Laplace and wave equations. Fourier transforms for the solution of p.d.e and their application to boundary value problems.

Learning Outcomes:

After the completion of the course, Students will be able to

- Classify partial differential equations and transform into canonical form
- Solve linear partial differential equations of both first and second order
- Apply partial derivative equation techniques to predict the behavior of certain phenomena.
- Apply specific methodologies, techniques and resources to conduct research and produce innovative results in the area of specialization.
- Extract information from partial derivative models in order to interpret reality.
- Identify real phenomena as models of partial derivative equations.

Recommended Books:

1. Evans, L. C. (2022). *Partial differential equations*. American Mathematical Society.
2. Borthwick, D. (2017). *Introduction to partial differential equations*. Springer.
3. Craig, W. (2018). *A course on partial differential equations*. American Mathematical Soc.
4. Buchanan, J. R., & Shao, Z. (2018). *A first course in partial differential equations*. World Scientific
5. Nandakumaran, A. K., & Datti, P. S. (2020). *Partial differential equations: classical theory with a modern touch*. Cambridge University Press.
6. Arrigo, D. (2023). *An Introduction to Partial Differential Equations*. Springer Nature.

Course Name: Rings and Fields	**Course Code:**MTH-523
Course Structure: Lectures: 3, Lab:0	**Credit Hours:** 3
Prerequisites: Group Theory and Linear Algebra	

<u>Specific Objectives of course:</u>

The course aims to provide the students with

- Well scientific knowledge of the theory of rings which is an important algebraic structure in mathematics specifically in algebra.
- Deep study of some interesting rings as Euclidean rings and a ring of polynomials over a field which is very useful in the study of finite fields and field extensions.
- These concepts have very important applications in Galois theory and especially for applications of Algebra.

<u>Course Outline:</u>

Definition of ring with examples, ring of continuous functions, Boolean ring,ring with unity, sub-ring with examples, direct product of rings, characteristics of ring,integraldomain., definition of ideal with examples, basic property of ideals, algebra of ideals,quotient ring, homomorphism of rings definition and basic properties, fundamental theoremof ideals,definition of modules with examples sub-modules, direct sum of modules quotient modules and homomorphism of modules.

<u>Learning Outcomes:</u>

- The students acquire basic concepts of a Ring as an algebraic structure like the definitions of a Ring, its Ideals, the factor ring, the auto morphisms of a ring, principal ring, prime and maximal ideals, the field of quotients of an integral domain, characteristic of a ring and direct sum of rings.
- Study the modules as another algebraic structure.
- Deep study of some important rings and field extensions which essentially arise from ideas of Galois Theory.

<u>Recom mended Books:</u>

1. Gouvêa, F. Q. (2012). *A guide to groups, rings, and fields* (Vol. 48). American Mathematical Soc..
2. Rowen, L. (2018). *Algebra: groups, rings, and fields*. CRC Press.
3. Shahriar, S. (2017). *Algebra in action: a course in groups, rings, and fields* (Vol. 27). American Mathematical Soc..
4. Jensen, C. U. (2022). *Model theoretic algebra with particular emphasis on fields, rings,*

modules. Routledge.

Course Name: Real Analysis-II	**Course Code:**MTH-522
Course Structure: Lectures: 3, Lab:0	**Credit Hours:** 3
Prerequisites: Real Analysis-I	

Specific Objectives of course:

A continuation of Real Analysis-I, this course aims to provide students

- with the specialist knowledge necessary for basic concepts in Real Analysis.
- More precisely, it strives to enable students to learn basic concepts about functions of bounded variation, grasp basic concepts about the total variation, learn about Riemann integrals, sequences and series of functions.

Course Outline:

Riemann Integrals and Riemann Integrals as limit of sum. Explicit and implicit functions. Limits of functions of several variables: Continuity of functions of partial derivatives of higher order, Jaccobians. Extreme values: Maxima and Minima.

Learning Outcomes:

- The students will be able to Learn the theory of Riemann integrals and Riemann integrals as a limit of sum, to be acquainted with the ideas of the total variation and to be able to deal with functions of bounded variation.
- The students will be able to develop a reasoned argument in handling problems about functions, especially those that are of bounded variation.
- The students will be able to develop the ability to reflect on problems that are quite significant in the field of real analysis and their functional relations.
- The students will be able to Learn the functions of several variables and their partial derivatives.

Recommended Books:

1. Stromberg, K. R. (2015). *An introduction to classical real analysis* (Vol. 376). American Mathematical Soc..
2. Bressoud, D. (2022). *A radical approach to real analysis* (Vol. 10). American Mathematical Society.
3. Ziemer, W. P., & Torres, M. (2017). *Modern real analysis* (Vol. 278). Springer.
4. Dshalalow, J. H. (2020). *Real analysis: an introduction to the theory of real functions and integration*. CRC Press.(Special Indian edition)

5. Bressoud, D. (2022). *A radical approach to real analysis* (Vol. 10). American Mathematical Society.

Course Name: Affine and Euclidean Geometry	**Course Code: MTH-525**
Course Structure: Lectures: 3	**Credit Hours: 3**
Prerequisites: Calculus-I	

Specific Objectives of course:

To familiarize mathematics students with the

- Axiomatic approach to geometry from a logical, historical, and pedagogical point of view
- Introduce them with the basic concepts of Affine Geometry, Affine spaces and Platonic Ployhedra.

Course Outline:

Vector spaces and affine geometry: Collinearity of three points, ratio AB/BC. Linear combinations and linear dependent set versus affine combinations and affine dependent sets. Classical theorems in affine geometry: Thales, Menelaus, Ceva, Desargues. Affine subspaces, affine maps. Dimension of a linear subspace and of an affine subspace. **Euclidean geometry**: Scalar product, Cauchy-Schwartz inequality: norm of a vector, distance between two points, angles between two non-zero vectors. Pythagoras theorem, parallelogram law, cosine and sine rules. Elementary geometric loci. **Orthogonal transformations:** Isometries of plane (four types), Isometries of space (six types). Orthogonal bases. **Platonic polyhedra**: Euler theorem on finite planar graphs. Classification of regular polyhedra in space. Isometries of regular polygons and regular polyhedra.

Learning Outcomes:

After successful completion of the course, students will be able To

- Identify key results about triangles and circles in Euclidean geometry.
- Write in a manner that further improves one's mathematical communication skills.
- Identify connections between linear algebra and transformational geometry.
- Develop an understanding of the fundamental concepts of axiomatic geometries.
- Identify the role of neutral geometry and the parallel postulate in the development of Euclidean geometry

Recommended Books:

1. E. Rees, *Notes on Geometry*, Springer, 2004.
2. M. A. Armstrong, *Groups and Symmetry*, Springer, 1998.
3. H. Eves, *Fundamentals of Modern Elementary Geometry*, Jones and Bartlett Publishers International, 1992
4. S. Stahl, *The Poincare Half-Plane A Gateway to Modern Geometry*, Jones and Bartlett Publishers International, 1993.

Course Name: Classical Mechanics	**Course Code: PHY-5**
Course Structure: Lectures: 3, Labs: 0	**Credit Hours:** 3
Prerequisites: Mechanics /None	

Course Objectives:

This course is a presentation of Newtonian mechanics at the intermediate level. Topics include dynamics of particles, and rigid bodies.

Course Outline: Electrostatics: Review of Newtonian Mechanics: Frame of reference, orthogonal transformations, angular velocity and angular acceleration, Newton's laws of motion, Galilean transformation, conservation laws, systems of particles, motion under a constant force, motions under variable force, time-varying mass system, The Lagrange Formulation of Mechanics and Hamilton Dynamics: Generalized co-ordinates and constraints, D'Alembert's principle and Lagrange's Equations, Legendre Transformation, Hamilton's principle, integrals of motion, non-conservative system and generalized potential, Lagrange's multiplier method, the Hamiltonian of a dynamical system, canonical equations, canonical transformations, Poisson brackets, phase space and Liouville's theorem. Central Force Motion: The two-body problem, effective potential, and classification of orbits, Kepler's laws, stability of circular orbits, hyperbolic orbits, and Rutherford scattering, center of the mass coordinate system, scattering cross-sections. Motion in Non-inertial Systems: Accelerated translational coordinate system, dynamics in the rotating coordinate system, motion of a particle near the surface of the earth. The Motion of Rigid Bodies: The Euler angles, rotational kinetic energy and angular momentum, the inertia tensor, Euler equations of motion, the motion of a torque-free symmetrical top, and stability of rotational motion.

Intended Learning Outcomes: At the end of the course the students will be able to Demonstrate the usefulness of these techniques in 'real world' problems, thereby Helping students to master other core subjects (classical mechanics, electrodynamics, and quantum mechanics). To develop problem-solving skills and a research attitude

Reference Material:

1. H. Goldstein, 2nd. Edn, (1980). 'Classical Mechanics', Addison Wesley, Reading, Massachusetts
2. T. M. Helliwell, V. V. Sahakian, (2020) "Modern Classical Mechanics", Cambridge University Press.
3. Jan Awrejcewicz, (2012)."Classical Mechanics: Kinematics and Statics", Springer Science & Business Media.
4. T. W. Kibble, Frank H. erkshire, 5th edition, (2004). "Classical Mechanics", Imperial College Press, 2004.
5. Choonkyu Lee, Hyunsoo Min, (2018)."Essential Classical Mechanics" World Scientific Publishing Company.

SEMESTER-VII

Course Name: Integral Equations	**Course Code:** MTH-614
Course Structure: Lectures: 3, Lab:0	**Credit Hours:** 3
Prerequisites: Ordinary Differential Equations	

Specific Objectives of course:

- To educate the students to gain insight into the application of advanced mathematics
- guide them through derivation of appropriate integral equations governing the behavior of several standard physical problems.

Course Outline:

Integal equation formulation of boundary value problems, classification of integral equations, method of successive approximation, HilbertSchmidt theory, Schmidt's solution of non-homogeneous integral equations, Fredholm theory, case of multiple roots of characteristic equation, degenerate kernels. Introduction to Wiener-Hopf technique.

Learning Outcomes:

- Student will be able to solve integral equations by different methods.
- Student will be able to recognize the applications of integral equations directly from setting up the physical relationship in a physical problem

- Students will be able to recall methods and techniques of solving integral equations.

Recommended Books:

1. C S. G. Mikhlin, linear integral equations Dover Publications, 2020
2. D. C. Sharma and M .C. Goyal, Integral Equations, Prentice Hall India Pvt. Limited, 2017.
3. Denis N Sidorovo, Samad Noeiaghdam, Integral equations theories, approximations and applications, MDPIAG, 2021.
4. Harendra Singh, Sudhanshu Aggarwal, Singular integral equations, 2021.
5. Ramakanta Meher, Numerical approximation of linear and nonlinear integral equations, central west publishing, 2021.

Course Name : Graph Theory	**Course Code:**MTH-613
Course Structure: Lectures: 3, Lab:0	**Credit Hours:** 3
Prerequisites: Set Theory, Mathematical Logic, Discrete Mathematics	

Specific Objectives of course:

The objectives of this course are to:

- Introduce the concepts of Graph and their different types as well as the isomorphism between them.
- Introduce the classes of Eulerian and Hamiltonian graphs, trees and weighted graphs.
- Illustrate how to find minimal walks in graphs.
- Introduce vertex and edge colorings of graphs.
- Find out the different methods of searching tree.

Course Outline:

Basic definitions, isomorphisms, walks, cycles and bipartite graphs, components, cut-edges, Eulerian graphs, vertex degrees and degree sequences, directed graphs, Eulerian digraphs, trees and distance, counting spanning trees and the matrix tree theorem, minimal spanning trees and shortest paths, matchings, Hall's theorem and coverings, maximum matchings, factors, cuts and connectivity, network flow problems, max-flow min-cut theorem, vertex colorings, bounds on chromatic numbers and Mycielski's construction, chromatic polynomials, chordal graphs, planar graphs, Euler's formula and Kuratowski's theorem, five and four color theorems.

Learning Outcomes:

After the completion of the course, Students will be able to

- Understand the theoretical base of the subject.
- Identify different types of the graphs and be able to apply different operations on them.
- Identify Eulerian and Hamiltonian graphs.
- Apply special algorithms to find minimal walks in weighted graphs.
- Apply special algorithms to find spanning trees in graphs.
- Find chromatic numbers and be able to find out planer graphs.

Recommended Books:

1. Reinhard Diestel, Graph theory, springer berlin Heidelberg, 2018.
2. Daniel A. Marcus, Graph theory, American mathematical society, 2020.
3. Beril Sirmacek, Graph theory advanced algorithms and applications, intechOpen, 2018.
4. Dr. A. Rahim Basha, Basick graph theory with applications, Sri Hariganesh Publications LLP, 2019
5. Jonathan L. Gross, jay Yellen, Ping zhang, 2nd edition, Hand book of graph theory, CRC Press LLC, 2017.

Course Name: Measure Theory-I	**Course Code:**MTH-611
Course Structure: Lectures: 3, Lab:0	**Credit Hours:** 3
Prerequisites: Real Analysis	

Specific Objectives of course:

- Revision of basic tools, including in particular the concept of countable/uncountable sets.
- Be able to describe at least one approach to the construction of Lebesgue measure, the Lebesgue integral of a function and measure spaces.
- Know the principal theorems as treated and their proofs and be able to use them in the investigation of examples.
- Be able to prove simple unseen propositions concerning measure spaces, Lebesgue measure and integration.
- To gain understanding of the abstract measure theory and definition and main properties of the integral.
- To construct Lebesgue's measure on the real line and in n-dimensional Euclidean space.
- To explain the basic advanced directions of the theory.

Course Outline:

Measure, outer measure, inner measure, Lebesgue measure, measurable sets, non-measurable sets, measurable functions, Lebesgue integral of a bounded functions, Lebesgue integral of arbitrary functions,Lebesgue integral and its relation to Riemann integral,convergence in measure, general convergence theorems, L_p -spaces, Stieltjes integral, product measure.

Learning Outcomes:

- Understand σ-algebras, measurable sets, measures, outer measures, Lebesgue measure and its properties, completion of measures.
- Understand measurable functions, approximation by simple functions.
- Understand Lebesgue integral, Monotone Convergence Theorem, Dominated Convergence Theorem, coincidence of Lebesgue and Riemann integral for Riemann integrable functions.
- Develop an appreciation of the basic concepts of measure theory. These methods will be useful for further study in a range of other fields, e.g. Stochastic calculus, Quantum Theory and Harmonic analysis.
- Establish relation to graduate attributes: The above outcomes are related to the development of the Science Faculty Graduate Attributes, in particular: Research, inquiry and analytical thinking abilities, Communication, and Information literacy

Recommended Books:

1. Evans, L. (2018). *Measure theory and fine properties of functions*. Routledge.
2. Kadets, V. (2018). *A course in functional analysis and measure theory*. Cham: Springer International Publishing.
3. Kadets, V. (2018). .*A course in functional analysis and measure theory*. Cham: Springer International Publishing.
4. Le Gall, J. F. (2022). *Measure Theory, Probability, and Stochastic Processes* (Vol. 295). Springer Nature.
5. Gigli, N. (2017). *Measure theory in non-smooth spaces*. De Gruyter Open.

Course Name: Mathematical Methods	**Course Code:**MTH-612
Course Structure: Lectures: 3, Lab:0	**Credit Hours:** 3
Prerequisites: calculus &ordinary differential equations	

Specific Objectives of course:

- The main objective of this course is to provide the students with a range of mathematical methods that are essential to the solutions of advanced problems encountered in the fields of applied physics and engineering.
- In addition, this course is intended to prepare the students with mathematical tools and techniques that are required in advanced courses offered in the applied physics and engineering problems.

Course Outline:

Laplace transform: Introduction and properties of Laplace transform, transforms of elementary functions, periodic functions, error function and Dirac delta function, inverse Laplace transform, convolution theorem, solution of ODEs by Laplace transform, Diffusion and wave equations**Hankeltransforms:** for the solution of PDEs and their application to boundary value problems. **Green's Functions and Transform Methods:** Expansion for Green's functions. Transform methods. Closed form Green's functions. **Perturbation Techniques:** Perturbation methods for algebraic equations. Perturbation methods for differential equations. **Variational Methods:** Euler-Lagrange equations. Integrand involving one, two, three and n variables. Special cases of Euler-Lagrange's equations. Necessary conditions for existence of an extremum of a functional. Constrained maxima and minima.

Learning Outcomes:

Upon successful completion of this course, students should be able to

- Implement basic operations in Fourier series and Laplace transforms.
- Be able to apply mathematical and computational methods to a range of problems in science and engineering.
- Be able to apply integral transforms to solve ordinary and partial differential equations.
- Be able to apply variational methods in solving variational problems.

Recommended Books:

1. Stephenson, G. (2020). *Mathematical methods for science students*. Courier Dover Publications.
2. Bayin, S. S. (2013). *Essentials of mathematical methods in science and engineering*. John Wiley & Sons.
3. Partee, B. B., ter Meulen, A. G., & Wall, R. (2012). *Mathematical methods in linguistics* (Vol. 30). Springer Science & Business Media.
4. Bleistein, N. (2012). *Mathematical methods for wave phenomena*. Academic Press.
5. Stephenson, G. (2020). *Mathematical methods for science students*. Courier Dover Publications.
6. Korevaar, J. (2014). *Mathematical Methods: Linear Algebra/Normed Spaces/Distributions/Integration* (Vol. 1). Elsevier.
7. Ammari, H., Bretin, E., Garnier, J., Kang, H., Lee, H., & Wahab, A. (2015). *Mathematical methods in elasticity imaging*. Princeton University Press.

SEMESTER-VII

Course Name: Functional Analysis	**Course Code:**MTH-624
Course Structure: Lectures: 3, Lab:0	**Credit Hours:** 3
Prerequisites: Real Analysis –I & II	

Specific Objectives of course:

- To know and be able to use the elementary properties of normed and inner product spaces.
- To be able to check whether a linear operator is bounded, to find its adjoint and determine whether operators are normal, self-adjoint, unitary or positive.
- To determine whether a bounded operator is invertible and understand the importance of the spectrum of a bounded linear operator.
- To study normed linear spaces and some of the linear operators between them and give some applications of their use.
- To introduce the theory of Lebesgue integration with the aim of providing examples of complete normed linear spaces of integrable functions.

Course Outline:

Banach Spaces: Definition and examples of normed spaces, Banach spaces, Characterization of Banach spaces. Bounded Linear Transformations: Bounded linear operators, Functionals and their examples, Various characterizations of bounded (continuous) linear operators, The space of all bounded linear operators, The open mapping and closed graph theorems, The dual (conjugate) spaces, Reflexive spaces. Hahn-Banach Theorem: Hahn-Banach theorem (without proof), Some important consequences of the Hahn-Banach theorem. Hilbert Spaces: Inner product spaces and their examples, The Cauchy-Schwarz inequality, Hilbert spaces, Orthogonal complements, The projection theorem, The Riesz representation theorem.

Learning Outcomes:

Upon completing the course, students will be able to:

- To learn to recognize the fundamental properties of normed spaces and of the transformations between them.
- Understand the notions of dot product and Hilbert space and apply the spectral theorem to the resolution of integral equations.
- Corelate Functional Analysis to problems arising in Partial Differential Equations, Measure Theory and other branches of Mathematics.

Recommended Books:

1. S. Malacride, Introduction to Functional Analysis, BookRix Publishing, 2023.

2. J.V. Neeren, Functional Analysis, Cambridge University Press Publishing, 2022.
3. T. Hytonen, J.V. Neerven, M. Veraar, L. Weis, Analysis in Banach Spaces Volume 2, Springer International Publishing, 2018.
4. M. Buntinas, Classical and Discrete Functional Analysis with Measure Theory, Cambridge University Press Publishing, 2022.
5. O.M. Shalit, A. First Course in Functional Analysis, 1st Edition, 2017.
6. S. Kesavan, Functional Analysis, 2nd Edition, 2023.
7. F.S. Botelho, Functional Analysis, Calculus of Variations and Numerical Methods for Models in physics and Engineering, 1st Edition, 2020.
8. P. Vaidyanathan, Functional analysis, Cambridge University Press Publishing, 2022.

Course Name: Measure Theory –II	**Course Code:**MTH-622
Course Structure: Lectures: 3, Lab:0	**Credit Hours:** 3
Prerequisites: Measure Theory-I	

<u>Specific Objectives of course:</u>

To familiarize mathematics students with the

- Lebesgue integral, Lebesgue integral for bounded functions, convergence theorems,
- the fundamental theorem of integral calculus, the Lebesgue integral for unbounded functions
- indefinite integrals mean convergence and the lebesgue classics L^p.

<u>Course Outline:</u>

This course includes Lebesgue integral,Lebesgue integral for bounded functions, convergence theorems, the fundamental theorem of integral calculus, the Lebesgue integral for unbounded functions, Derivatives, Non differentiable functions of bounded variations of absolute continuous functions, indefinite integrals mean convergence and the lebesgue classics L^p.

<u>Learning Outcomes:</u>

After successful completion of the course, students will be able to

- understand the fundamentals of measure theory and be acquainted with the proofs of the fundamental theorems underlying the theory of integration.
- will also have an understanding of how these underpin the use of mathematical concepts such as volume, area, and integration and they will develop a perspective on the broader impact of measure theory.

- have the ability to pursue further studies in this and related areas.

Recommended Books:

1. A. Khanfer, Measure Theory and Integration, Springer Publishing Company, 2023.
2. Jean-Francois Le Gall, Measure Theory, Probability and Stochastic Processes, Springer International Publishing, 2022.
3. M. Buntinas, Classical and Discrete Functional Analysis with Measure Theory, Cambridge University Press Publishing Company, 2022.
4. F. Smarrazzo, A. Tesei, Measure Theory and Nonlinear Evolution Equations, De Gruyter Publishing Company, 2022.
5. S. Axler, Measure, Integration and Real Analysis, Springer International Publishing, 2019.
6. A. Gavrilut, L. Merches, M. Agop, Atomicity through Fractal Measure Theory, Springer International Publishing, 2019.
7. S. Shirali, A Concise Introduction to Measure Theory, Springer International Publishing, 2019.
8. S. Kesavan, Measure and Integration, Springer Nature Singapore Publishing, 2019.

Course Name : Fluid Mechanics	**Course Code:**MTH-621
Course Structure: Lectures: 3, Lab:0	**Credit Hours:** 3

Prerequisites: Knowledge of Calculus-I, II , Ordinary and Partial Differential Equations.

Specific Objectives of course:

- To help understanding the nature of fluid statics, in particular dealing with problems related to hydrostatic forces.
- To be able to analyze the problems related to elementary fluid dynamics especially for incompressible flows using Bernoulli equation in particular.
- To learn the basic models for Inviscid and viscous fluid flow using control volume and differential analysis approaches.
- To develop the understanding by applying mathematical models to simple realizable configurations along with practical considerations.
- To apprehend the applications/solutions of models developed in the advanced course in industrial applications using analytical as well as numerical methods.

Course Outline

Introduction: Dimensions, units and physical quantities, gases and liquids, pressure and temperature, properties of fluids, thermodynamics properties and relationship. Pressure variation. Fluid statics: Pressure variation, forces on plane and curved surfaces. Fluids in Motion: Lagrangian and Eulerian description, pathlines, streaklines and steamlines, acceleration, angular velocity and vorticity, classification of fluid flows, Bernoulli's equation. Navier Stokes Equations: The integral and differential forms of the conservation of mass, momentum and energy. Viscous Flow: Incompressible viscous flow using Navier-Stokes equations, pipe flow, boundary layers, separation, introduction to turbulence.

Learning Outcomes:

Upon the successful completion of course, students will be able to:

- Explain the key fluid properties used in the analysis of fluid behavior.
- Compute the pressure and hydrostatic pressure force at various locations in a fluid.
- Apply the Bernoulli, continuity and energy equations to solve simple fluid flow problems.
- Analyze different fluid flow models using finite control volume and differential analysis approaches.

Recommended Books:

1. Spurk, H.Joseph and Nuri Aksel, *Fluid Mechanics*, 3^{rd} Edition, Cham: Springer International Publishing, 2020.
2. Ron Darby, P.Raj Chhabra, Chemical Engineering Fluid Mechanics, 3^{rd} Edition, CRC Press publishing, 2022.
3. R. K. Rajput, A Textbook of Fluid Mechanics, 1^{st} Edition, S. Chand Publishing, 2019.
4. W. Robert Fox, W. John. Mitchell, T.Alan McDonald, Fox and McDonald's Introduction to Fluid Mechanics, 10^{th} Edition, Wiley publishing company, 2020.
5. Jure Ravnik, L. Škerget, Santiago Hernández, Advances in Fluid Mechanics XII, 12^{th} Edition, WIT Press publishing, 2018.
6. Dia Zeidan, Jochen Merker, Eric Goncalves Da Silva, T. Lucy Zhang, Numerical Fluid Dynamics, 1^{st} Edition, Springer Singapore publisher, 2023.
7. Naseem Uddin, Fluid Mechanics A Problem-Solving Approach , 1^{st} Edition, CRC Press, 2022.

Course Name: Mathematical Statistics	**Course Code: MTH-612**
Course Structure: Lectures: 3, Lab:0	**Credit Hours:** 3
Prerequisites: Real Analysis-I	
Specific Objectives of course:	

The objectives of this course are to:

- Understand the theory of statistics and their applications.
- Understand the concepts of Probability.
- To analyse different distributions along with their properties.
- Aware the students about mathematical expectation, variance, moment generating function and moment about mean & about origin.
- To understand the concepts of testing hypothesis.

Course Outline:

Statistical inference. Maximum likelihood estimators. Properties of maximum likelihood estimators. Sufficient statistics. Jointly sufficient statistics. Minimal sufficient statistics. The sampling distribution of a statistic. The chi square distribution. Joint distribution of the sample mean and sample variance. That distribution. Confidence intervals. Unbiased estimators. Fisher information. Testing simple hypotheses. Uniformly most powerful tests. The t test. The F distribution. Comparing the means of two normal distributions. Tests of goodness of fit. Contingency tables. Equivalence of confidence sets and tests. Kolmogorov- Smirnov tests. The Wilcoxon Signed-ranks test. The Wilcoxon-Mann-Whitney Ranks test.

Learning Outcomes:

After the completion of the course, Students will be able to

- Differentiate between discrete and continuous random variables.
- Solve the problems related to Bay's theorem.
- Calculate mean, variance, Standard deviation of different types of theoretical distribution.
- Apply different types of tests of significance.
- Differentiate between types of error. 6. Understand Null & alternative hypothesis for testing.

Recommended Books:

1. S.C. Gupta, V.K. Kapoor, Fundamentals of Mathematical Statistics, 12^{th} Edition, Sultan Chand & Sons publishing, 2020.
2. Dieter Rasch, Dieter Schott, Mathematical Statistics, Wiley and sons publishing, 2018.
3. Fetsje Bijma, Marianne Jonker, A. W. van der Vaart, An Introduction to Mathematical Statistics, Amsterdam University Press publisher, 2017.
4. Barbara Illowsky, Susan Dean, Openstax, Introductory Statistics, Open Stax Textbooks publisher, 2022.
5. Daniel Hernández-Hernández, Florencia Leonardi, Juan Carlos Pardo Millán, H. Ramsés Mena, Advances in Probability and Mathematical Statistics, Springer International

Publishing,2021.

ELECTIVE/OPTIONAL COURSES

Course Name: Differential Geometry	**Course Code: MTH-631**
Course Structure: Lectures: 3,Lab:0	**Credit Hours:** 03
Prerequisites: Calculus-I	

Specific Objectives of course:

- To get introduced to the concept of a regular parameterized curve
- To Understand the concept of curvature of a space curve and signed curvature of a plane curve.
- To be able to understand the fundamental theorem for plane curves.
- To get introduced to the notion of Serret-Frenet frame for space curves and the involutes and evolutes of space curves with the help of examples.
- To be able to compute the curvature and torsion of space curves.
- To be able to understand the fundamental theorem for space curves.
- To get introduced to the concept of a parameterized surface with the help of examples.
- To understand the idea of orientable/non-orientable surfaces.

Course Outline:

Theory of Space Curves: Introduction, index notation and summation convention. Space curves, arc length, tangent, normal and binormal. Osculating, normal and rectifying planes. Curvature and torsion. The Frenet-Serret theorem. Natural equation of a curve. Involutes and evolutes, helices. Fundamental existence theorem of space curves.**Theory of Surfaces:** Coordinate transformation. Tangent plane and surface normal. The first fundamental form and the metric tensor. The second fundamental form. Principal, Gaussian, mean, geodesic and normal curvatures. Gauss and Weingarten equations. Gauss andCodazzi equations.

Learning Outcomes:

After the completion of the course the student

- can examine the length and parametrization of curves
- masters the definitions of curvature and torsion and can apply these
- knows the local canonical form of curves

- knows the contents and the significance of the Jordan curve theorem and the isoperimetric inequality.
- can examine the properties of surfaces using different expressions for surfaces
- knows the first fundamental form of surfaces
- can determine areas and various curvatures of surfaces
- knows the contents and the significance Gauss' Theorema egregium

Recommended Books:

1. Banchoff, T. F., & Lovett, S. (2022). *Differential geometry of curves and surfaces*. CRC Press.
2. Umehara, M., & Yamada, K. (2017). *Differential geometry of curves and surfaces*.
3.
4. Do Carmo, M. P. (2016). *Differential geometry of curves and surfaces: revised and updated second edition*. Courier Dover Publications.
5. Kolár, I., Michor, P. W., & Slovák, J. (2013). *Natural operations in differential geometry*. Springer Science & Business Media.
6. Tapp, K. (2016). *Differential geometry of curves and surfaces*. Berlin: Springer.

Course Name: Optimization theory	**Course Code:MTH-632**
Course Structure: Lectures: 3, Lab:0	**Credit Hours:** 3
Prerequisites: Numerical Analysis	

Specific Objectives of course:

The subject of optimization can be studied as a branch of pure mathematics and has application in nearly all the branches of science and technology. Therefore this course aims to equip students from those aspects of optimization methods which are of importance in real life problem solving.

Course Outline:

Basic Results:

Definition, Condition for unconstrained variables, Equality constraints, General consideration and necessary conditions of Inequality constraints. Convexity, abnormal point and sufficient conditions for Inequality. Sadle point condition and Duality. **Unconstrained Optimization:** Line search Methods, General search methods, Gradient Methods, Newton and Quasi Newton Methods.**Linear Programming:**Solution of LP Problem, Duality. **Constrained Optimization:** General Properties of the solution, Projection Methods, Quadratic Programming, Application of projection methods to nonlinear constraints.

Learning Outcomes:

After the completion of course students will be able to:

- Recognize, formulate, and solve linear programming problems
- Understand the simplex method for linear programming
- Learn nonlinear programming with constraints and no constraints
- Understand multi-objective optimization and be able to generate Pareto
- Be able to model complex systems using surrogate modeling and design space reduction techniques

Recommended Books:

1. Chong, E. K. P., Lu, W., Żak, S. H. (2023). *An Introduction to Optimization: With Applications to Machine Learning.* United Kingdom: Wiley.
2. Yong, J. (2018). *Optimization Theory: A Concise Introduction*. Singapore: World Scientific Publishing Company.
3. Rao, S. S. (2019). *Engineering optimization: theory and practice*. John Wiley & Sons.
4. Gill, P. E., Murray, W., Wright, M. H. (2020). *Practical Optimization*. United States: Society for Industrial and Applied Mathematics.
5. Calafiore, G. C., El Ghaoui, L. (2018). *Optimization Models*. United Kingdom: Cambridge University Press.

Course Name: Algebraic Topology	**Course Code:** MTH-633
Course Structure: Lectures: 3, Lab:0	**Credit Hours:** 3
Prerequisites: Set Topology and Linear Algebra	

Specific Objectives of course:

To investigate spaces by mapping them to algebraic objects such as groups, and thereby bring into play new methods and intuitions from algebra to answer topological questions.

Course Outline:

CW-complexes, delta-complexes, simplicial homology, exact sequences, diagram chasing, singular homology, homotopies and chain homotopies, categories and functors, Eilenberg-Steenrod axioms, excision, computations for spheres, equivalence of simplicial and singular homology, cellular homology, Mayer-Vietoris sequences, the Mayer-Vietoris argument, homology with coefficients, tensor products, tor, universal coefficient theorem for homology, products of simplices, the Eilenberg-Zilber shuffle "product" map, diagonal approximations, the Alexander-Whitney map, method of acyclic models, Kunneth formula, duality, cohomology, ext, universal coefficients for cohomology, projective spaces and Grassmannians, cup products, relative cup products, dual Kunneth formula, field coefficients, cup products in cohomology of projective spaces, manifolds, local orientations, global orientations, cap products and choices of appropriate sign conventions, statement of Poincare duality, limits, compactly supported cohomology, proof of Poincare duality, finish proof of Poincare duality, intersection pairing and cup product, Lefschetz fixed point theorem, finish proof of Lefschetz theorem.

Learning Outcomes:

- Recognize, classify and construct surfaces. - Compute on simple examples classical invariants of algebraic topology : fundamental group, Euler class, homology group.
- Deduce some topological properties of spaces from invariants of algebraic topology.
- Develop in detail an element of algebraic topology theory.

Recommended Books:

1. Jie Wu, Mahender Singh, Yongiin Song., Algebraic Topology And Related Topics, 2019, Springer Basel AG.
2. Bray C., Butscher A., Salzedo S., Algebraic Topology, 2021,
3. Adhikari R. M., Basic Topology 3: Algebraic Topology and Topology of Fiber Bundles., 2023, Springer Nature Singapore.
4. Miller H., Lectures on Algebraic Topology, 2022, World Scientific.
5. Kammeyer, H., Introduction to Algebraic Topology, 2022, Springer International Publishing.

Course Name: Galois Theory	**Course Code:**MTH-634
Course Structure: Lectures: 3, Lab:0	**Credit Hours:** 3
Prerequisites: Linear Algebra, Group Theory, Rings and Fields.	

Specific Objectives of course:

To educate the students with the problem of solutions of polynomial equations both in explicit terms and in terms of abstract algebraic structures; to demonstrates the tools of abstract algebra (linear algebra, group theory, rings and ideals) as applied to a meaningful problem.

Course Outline:

Integral domains and fields, homorphisms and ideals, quotient rings, polynomial rings in one indeterminate over fields, prime ideals and maximal ideals, irreducible polynomials, algebraic and transcendental field extensions, simple extensions, composite extensions, splitting fields, the degree of extension, ruler and compass constructions, normality and separability, circle division, the Galois group, toots of unity, solvability by radicals, Galois extensions, the fundamental theorem of Galois theory, Galois's great theorem, algebraically closed fields.

Learning Outcomes:

After the completion of course students will be to:

- Calculate groups of low degree Galois equations and deduce their resolvability by radicals.
- Construct quotient groups and rings and finite bodies and operate within them.
- Manipulate expressions involving algebraic and transcendent elements.
- Relate geometric constructions with algebraic extensions.

Recommended Books:

1. Stewart, I. (2022). *Galois theory*. CRC press.
2. Garling, D. J. (2021). *Galois theory and its algebraic background*. Cambridge University Press.
3. Douady, R., & Douady, A. (2020). *Algebra and Galois theories*. Springer International Publishing.
4. Bewersdorff, J. (2021). *Galois theory for beginners: a historical perspective* (Vol. 95). American Mathematical Soc..
5. Sinha, R. (2020). *Galois theory and advanced linear algebra*. Springer.
6. Coleman, R., & Zwald, L. (2023). Algebraic Number Theory with Elementary Galois Theory.

Course Name : Riemannian Geometry	**Course Code:**MTH-635
Course Structure: Lectures: 3, Lab:0	**Credit Hours:** 3
Prerequisites: Algebra-I, Calculus-I, Set Topology	

Specific Objectives of course:

- To introduce the concept of a manifold as a space with a locally Euclidean smooth structure.
- To examine Riemann's notion of an intrinsic method for measuring distance on a manifold.
- To study the curvature and geodesics of Riemannian manifolds and obtain some geometric consequences.

Course Outline

Definition and examples of manifolds, sub manifolds, smooth maps, tangents, coordinate vector fields, tangent spaces, dual spaces, algebra of tensors, vector fields, tensor fields, integral curves, affine connections and Christoffel symbols, covariant differentiation of tensor fields, geodesics equations, curve on manifold, parallel transport, lie transport, lie derivatives and lie brackets, geodesic deviation, differential forms, introduction to integration theory on manifolds, Riemannian curvature tensor, geodesics, exponential map, curvature and examples,completeness and Hopf-Rinowtheorem, manifolds with constant curvature, sphere, geometry of hyperbolic space.

Learning Outcomes:

- The concept of a manifold, the intrinsic idea of tangent vector fields and differential 1-forms, and how these provide a framework for differential calculus with many applications (for example in General Relativity.
- The notion of a Riemannian metric, and how it generalises the first fundamental form of surfaces in Euclidean space.
- Connections (or covariant derivatives), parallel transport and curvature, and how to apply them in Riemannian geometry.
- The local theory of geodesics, and their interaction with the global structure of the manifold.
- Simple aspects of the notion of Riemannian curvature.

Recommended Books:

1. Beggs, E. J., & Majid, S. (2020). *Quantum Riemannian Geometry* (Vol. 355). Berlin: Springer.

1. Berestovskii, V., & Nikonorov, Y. (2020). *Riemannian manifolds and homogeneous geodesics*. Cham: Springer.
2. Lee, D. A. (2021). *Geometric relativity* (Vol. 201). American Mathematical Society.
3. Gallier, J. Q., & Quaintance, J. (2020). *Differential geometry and lie groups* (Vol. 12). New York, NY, USA: Springer International Publishing.
4. Ou, Y. L., & Chen, B. Y. (2020). *Biharmonic submanifolds and biharmonic maps in Riemannian geometry*.
5. Jost, J., & Jost, J. (2008). *Riemannian geometry and geometric analysis* (Vol. 42005). Berlin.

Course Name: General Relativity	**Course Code:**MTH-636

Course Structure: Lectures: 3, Lab:0	**Credit Hours:** 3
Prerequisites: Special Relativity	

Specific Objectives of course:

To familiarize the students with theories presented in general relativity and explain the predictions of this theory; to discuss physical and mathematical aspects of general relativity.

Course Outline

Vectors, one forms and the metric, manifolds, parameterized curves, tangent vectors, vectors in curved space, the metric tensor and covariant differentiation, the curvature, Ricci and Weyl tensors, curves in manifolds, parallel transport, Geodesics, Bianchi identity, Lie derivative and isometries, energy momentum tensor, killing vectors, brief literature review of symmetries in general relativity.

Learning Outcomes

Upon successful completion, students will have the knowledge and skills to:

- Demonstrate understanding of the mathematics underpinning manifolds, tensors, metrics, geodesics and the Riemann tensor.
  Prove fundamental properties of, and relationships between, those mathematical elements;
- Derive geodesics from a given metric, and derive metrics from the Einstein field equation for simple forms of the stress-energy tensor;
  Describe the fundamental properties of gravitational waves;
  Evaluate methods to detect gravitational waves on Earth.

Recommended Books:

1) James B, Hartle., Gravity: An Introduction to Einstein's General Relativity, 2022, Cambridge university press.
2) Ryder, L., Introduction to General Relativity, 2020, Cambridge university press.
3) Jetzer, P., Application of General Relativity, 2022, Springer International Publishing.
4) Dhurandhar, S., Mithra, S., General Relativity and Gravitational Waves, 2022, Springer International Publishing.
5) Ronald J. Adler., General Relativity and Cosmology, 2022, Springer International Publishing.

Course Name: Dynamical Systems	**Course Code:**MTH-637
Course Structure: Lectures: 3, Lab:0	**Credit Hours:** 3
Prerequisites: Linear Algebra and Calculus	

Specific Objectives of course:

To understand the fundamental concepts of the dynamical systems; to solve representative problems in one dimensional and higher dimensional dynamics.

Course Outline

One Dimensional Dynamics: Elementary definitions of dynamical systems, hyperbolicity, the quadratic family, symbolic dynamics, structure stability, chaos, bifurcation theory. Higher Dimensional Dynamics: The dynamics of linear maps, attractors, the stable and unstable manifold theorems, global results and hyperbolic sets, periodic points.

Learning Outcomes:

- To introduce students to the basic mathematical skills for the qualitative solving of low dimensional systems of ordinary differential equations in continuous time, including dimensionless forms, phase portraits, and bifurcations.
- To provide a brief introduction to the way ordinary differential equations can be used to model, explain and interpret real world problems.
- To provide a brief introduction to the theory and concepts that underpin the field of dynamical systems.

Recommended Books:

1. Devaney, R. (2018). *An introduction to chaotic dynamical systems*. CRC press.
2. Lynch, S. (2018). *Dynamical systems with applications using python*. Switzerland: Springer International Publishing.
3. Jalili, N., Candelino, N. W. (2023). *Dynamic Systems and Control Engineering*. United Kingdom: Cambridge University Press.
4. Layek, G. (2019). *An Introduction to Dynamical Systems and Chaos*. India: Springer India.
5. Feldman, D. P. (2019). *Chaos and Dynamical Systems.* United Kingdom: Princeton University Press.
6. Barreira, L., Valls, C. (2019). *Dynamical Systems by Example.* Germany: Springer International Publishing.

Course Name: Quantum Mechanics	**Course Code:**MTH-638
Course Structure: Lectures: 3, Lab:0	**Credit Hours:** 3
Prerequisites: Mechanics	

Specific Objectives of course:

This Course Enables the Student to

- Understand the linear equations, vector spaces, matrices, linear transformations, determinants, eigenvalue, eigenvectors, etc.
- Learn to use Laplace transform methods to solve differential equations.
- Introduce the Fourier series and its application to the solution of partial differential equations

Course Outline:

Inadequacy of classical mechanics, black body radiation, photoelectric effect, comption effect, Bohr's theory of atomic structure, wave-particle duality, De-Broglies postulate, the uncertainty principle, uncertainty of position and momentum, statement and proof of the uncertainty principle, energy-time uncertainty, eigenvalues and eigenfunctions, operators and eigenfunctions, linear operators, operators formulism in quantum mechanics, orthonormal system, hermitian operators and their properties, simultaneous eigen-functions, parity operators, postulate of quantum mechanics, schroedinger wave, equation motion in one dimension, step potential, potential barrier, potential well, harmonic oscillator,motion in three dimensions, angular momentum, Pauli exclusion principle, hydrogen atom. heisenberg equations of motion and equivalence of Schrodinger and Heisenberg physical pictures, scattering theory, Born approximation, partial wave analysis, optical theory, time dependent & time independent perturbation theory, selection rules. Klein-Gordon equation, Dirac's equation, spin angular momentum.

Learning Outcomes:

Upon successful completion of this course it is intended that a student will be able to:

- Students will demonstrate competence with the basic ideas of linear algebra including concepts of linear systems, independence, theory of matrices, linear transformations, bases and dimension, eigenvalues, eigenvectors and Diagonalization.
- Use the method of Laplace transforms to solve initial-value problems for linear differential equations with constant coefficients.
- Solve a Cauchy problem for the wave or diffusion equations using the Fourier Transform

Recommended Books:

1. Norsen, T. (2017). *Foundations of quantum mechanics*. Springer.
2. Davies, P. C. (2018). *Quantum mechanics*. Routledge.
3. Izaac, J., & Wang, J. (2018). *Computational quantum mechanics*. Berlin: Springer.
4. Von Neumann, J. (2018). *Mathematical foundations of quantum mechanics: New edition* (Vol. 53). Princeton university press.
5. Cohen-Tannoudji, C., Diu, B., & Laloë, F. (2019). *Quantum mechanics, volume 3:*

fermions, bosons, photons, correlations, and entanglement. John Wiley & Sons.
6. Dürr, D., & Lazarovici, D. (2020). *Understanding quantum mechanics*. Springer: Berlin/Heidelberg, Germany.
7. Kastner, R. E. (2022). *The Transactional Interpretation of Quantum Mechanics: A Relativistic Treatment*. Cambridge University Press.

Course Name : Lie Groups and Lie Algebras	**Course Code:**MTH-639
Course Structure: Lectures: 3, Lab:0	**Credit Hours:** 3
Prerequisites: Fundamental Concepts of Group Theory and Differential Geometry	

Specific Objectives of course:

To learn the structure of Lie groups, Lie algebras and their (complex) representations.

Course Outline

Definition of Lie group and Lie algebra, the exponential mapping, matrix Lie groups, complex Lie groups, infinite dimensional Lie groups, Cartan's theorem on closed subgroups, the adjoint representation, universal covering groups, the universal enveloping algebra, compact Lie groups, representation of Lie groups on finite dimensional vector space and on Hilbert space, Lie Subgroups, properties of Lie algebra, Lie sub algebra, actions of Lie groups and Lie algebras, structure constants, direct sums, Lie algebra of matrix Lie groups, universal enveloping algebras.

Learning Outcomes:

After the completion of course students will be able to:

- An understanding of matrix groups and their Lie algebras, as an application of linear algebra.
- An understanding of transformation groups and the notion of invariance.
- To gain an appreciation of the interplay between different areas of mathematics in the setting of matrix groups.

Recommended Books:

1. B. Hall, Second Eidtion, Lie Groups, Lie Algebras, and Representations, An Elementary Introductio, Germany: Springer Int. Publishing, (2016).

2. R. Gilmore, Lie Groups, Lie Algebras, and Some of Their Applications. United States: Dover Publications, (2012).
3. V. Varadarajan, Lie Groups, Lie Algebras, and Their Representations. Switzerland: Springer New York. (2013).
4. V. Gorbatsevich, E. Vinberg, Lie Groups and Lie Algebras I: Foundations of Lie Theory Lie

Transformation Groups. Germany: Springer Berlin Heidelberg, (2013).

5. J. Gallier, J. Quaintance, Differential Geometry and Lie Groups: A Computational Perspective. Germany: Springer Int. Publishing, (2020).
6. A. Kirillov, Jr, A. A. Kirillov, An Introduction to Lie Groups and Lie Algebras. United Kingdom: Cambridge University Press, (2017).

Course Name: Special Relativity	**Course Code:**MTH-640
Course Structure: Lectures: 3, Lab:0	**Credit Hours:** 3
Prerequisites: Electricity and Magnetism	

Specific Objectives of course:

- Understand the motivation for developing the Theory of Special Relativity.
- Understand Einstein's postulates and their consequences.
- Understand how to apply Einstein's postulates to describe simultaneity.
- Understand how to model length contraction and time dilation.
- Understand how to apply Lorentz transformations and make space-time diagrams.
- Understand how to model the energy and momentum of a relativistic object.

Course Contents:

Development of the pre-Newtonian and Newtonian theories of motion. Einstein's Special theory of Relativity: length contraction, time dilation and simultaneity, velocity addition for 1-d motion, the extension of special relativity to 3-dimensions, invariant quantities and tensors, coordinate transformations, the 4-vector formulation of special relativity, and its geometric and group aspects. Physical Applications of Special Relativity: Doppler effect, Compton effect, particle scattering, particle production, decay and binding energy, use of 4-vector formulation for electromagnetism and its consequences gauge transformations and gauge groups, special relativity with small accelerations and its geometrical implications.

Learning Outcomes:

After completion of this course, student will be able to

- Demonstrate knowledge and broad understanding of Special Relativity.
- Explain the meaning and significance of the postulate of Special Relativity.
- Explain true nature of Lorentz transformation and Doppler effect.
- Recall the setup and significance of Michelson-Morley experiment.
- Explain relativistic momentum and Einstein field equations.

Recommended Books:

1. Dragan, A., Unusually Special Relativity, 2022, World Scientific Publishing

Europe Limited.
2. Thomas Strohm., Special Relativity for the Enthusiast, 2023, springer international Publishing.
3. Susskind, L,. Friedman, A., Special Relativity and Classical Field Theory, 2019, Basic books.
4. Deshko, Y., Special Relativity for Inquiring Minds, 2022, Springer International Publishing.
5. Rahaman, F., A Special Theory of Relativity A Mathematical Approach, 2022, Springer Nature Singapore.
6. Rafelski, J., Modern Special Relativity, 2022, springer international Publishing.

Course Name: Convex Analysis	**Course Code:**MTH-641
Course Structure: Lectures: 3, Lab:0	**Credit Hours:** 3
Prerequisites: Calculus-I	

<u>Specific Objectives of course:</u>

To know about theory of convex functions, convex sets and some results for convex functions.

Course Outline

Convex set, J- convex function, convex and log- convex function, continuity and differentiability of convex function, epigraph of convex function, relation between convex and J- convex functions, characterizations, differences of convex functions, affine function, sub differential of convex function, support line of convex functions, conjugate convex functions, affine sets, convex and affine hull.

<u>Learning Outcomes:</u>

After completion of course students will be able to:

- understand the concept of Convex Analysis, convex sets, convex functions, Differential of the convex function.
- Developing ability to study the Hadamard-Hermite inequalities and their applications.
- Prepare students to be self independent and enhance their mathematical ability by giving them home work and projects.

<u>Recommended Books:</u>

1. Mordukhovich, B., & Nam, N. M. (2023). *An easy path to convex analysis and applications*. Springer Nature.
2. Magaril-Il′yaev, G. G., & Tikhomirov, V. M. (2003). *Convex analysis: theory and applications* (Vol. 222). American Mathematical Soc..

3. Correa, R., Hantoute, A., & López, M. A. (2023). *Fundamentals of convex analysis and optimization: a supremum function approach*. Springer Nature.
4. Hiriart-Urruty, J. B., & Lemaréchal, C. (2013). *Convex analysis and minimization algorithms I: Fundamentals* (Vol. 305). Springer science & business media.
5. Mordukhovich, B. S., & Nam, N. M. (2022). *Convex analysis and beyond : Volume I: Basic theory*. Springer Nature.

Course Name: Dynamics	**Course Code:**MTH-642
Course Structure: Lectures: 3, Lab:0	**Credit Hours:** 3
Prerequisites:	

Specific Objectives of course:

- Develop an understanding of the principles of dynamics.
- Develop an ability to analyze problems in a systematic and logical manner, including the ability to draw free-body diagrams of rigid body.
- Ability to analyze the dynamics of rigid body.
- Discuss the motion on smooth and rough planes.
- Discuss general motion of rigid body, Keplers laws.

Course Outline:

Particle Dynamics: Projectile motion under gravity, constrained particle motion, angular momentum of a particle. Orbital Motion: Motion of a particle under a central force, use of reciprocal polar co-ordinates, use of pedal co-ordinates and equations, Kepler's laws of planetry motion. Motion of a system of Particles: Linear momentum of a system of particles, angular momentum and rale of change of angular momentum of a system, use of centroid, moving origins, impulsive forces, elastic impact. Introduction to Rigid Body Dynamics: Moments and products of inertia, the theorems of parallel and perpendicular axes, angular momentum of a rigid body about a fixed point and about fixed axes, principal axes. Kinetic energy of a rigid body rotating about a fixed point, general motion of a rigid body, momental ellipsoid, equimomental system, coplanar distribution.

Learning Outcomes:

After the completion of the course, Students will be able to

- An ability to construct free-body diagrams.
- An understanding of the analysis of distributed loads.
- A knowledge of internal forces and moments in members.
- Apply Keplers laws to solve the problems.

Recommended Books:

1. Ghosh, A. (2018). *Introduction to Dynamics. Germany*. Springer Nature Singapore.
2. Ghosh, A. (2019). *Introduction to Dynamics*. Singapore. Springer Nature Singapore.
3. Schmerr, L. W. (2019). *Engineering Dynamics 2.0*. Fundamentals and Numerical Solutions. Switzerland: Springer International Publishing.
4. Hoiles, W., Krishnamurthy, V., Cornell, B. (2018). *Dynamics of Engineered Artificial Membranes and Biosensors*. United Kingdom: Cambridge University Press.
5. Meriam, J. L., Kraige, L. G., Bolton, J. N. (2020). *Engineering Mechanics: Dynamics*. United Kingdom: Wiley.

Course Name : Computational Fluid Dynamics	**Course Code:**MTH-643
Course Structure: Lectures: 3, Lab:0	**Credit Hours:** 3

Prerequisites: Knowledge of Calculus-I & II, Ordinary and Partial Differential Equations, Fluid Mechanics, Numerical Analysis, Mat Lab or Fortran or C.

Specific Objectives of course:

To enable the students to

- apply numerical techniques of finite difference method to solve partial differential equations related to fluid dynamics
- to learn computer programmes for numerical solutions of partial differential equations related to flow phenomena
- to enable the students to sketch and analyze various solutions based on the output of the computer programmes.

Course Outline:

A brief review of NavierStok's equations, numerical methods for modeling parabolic and elliptic equations,model equations, discretization of derivatives with finite differences, finite difference method]] for parabolic equations, explicit method with various boundary conditions, implicit methods, Crank Nicolson and Keller box schemes with examples and programming, finite difference methods for hyperbolic partial differential equations with examples and computer programs, the laminar boundary layer flow over rotating disk with programming and visualizations.

Learning Outcomes:

Upon completion of the course, students will be able to:

- Use numerical methods for solving various fluid and heat transfer problems.

- Have better understanding of fluid mechanics and heat transfer
- Formulate steady and unsteady Finite-Difference & Finite-Volume numerical methods and develop solution algorithms.
- Program and simulate simple CFD problems
- Understand the CFD role in industrial design applications and its limitations

Recommended Books:

1. Sreenivas Jayanti, Computational fluid Dynamics for Engineers and Scientists,1st Edition, Springer Netherlands Publisher, 2018
2. Oleg Zikanov, Essential Computational Fluid Dynamics, 2nd Edition, Wiley Publisher, 2019
3. Meng Eai Woo, Computational Fluid Dynamics Simulation of Spray Dryes, 1st Edition, CRC Press, 2019
4. Vijay K. Garg, Applied Computational Fluid Dynamics, 1st Edition, CRC Press, 20219
5. George Qin, Computational Fluid Dynamics for Mechanical Engineering, 1st Edition, CRC Press, 2021
6. Pradip Majumdar, Computational Fluid Dynamics and Heat Transfer, 2nd Edition, CRC Press, 2022
7. Jiyuan Tu et al, Computational Fluid Dynamics A Practical Apprroach, 4th Edition, Elsevier Sci. Publisher, 2022

Course Name: Mathematical Modeling &Simulations	**Course Code:**MTH-644
Course Structure: Lectures: 3, Lab:0	**Credit Hours:** 3

Prerequisites: Partial Differential Equations, Programming in MATLAB

Specific Objectives of course:

The main objective of this module to gain the knowledge about system and its behavior so that a person can transform the physical behavior of a system into a mathematical model that can in turn transform into a efficient algorithm for simulation purpose.

Learning Outcomes:

The students will be able to

- Define a model and modelling.
- Explain when to and why we use models

- Describe the modelling process
- Describe different types of Models.
- Explain Simulation
- State why we need simulation
- Describe how simulations are done
- Describe various types of Simulations. Give examples of Simulation
- Show areas of applications of Simulation
- Can learn different types of Models
- Differentiate between Visual and Conceptual.

Course Outline:

Concept of model, modeling and simulation, functions, linear equations, linear differential equations, nonlinear differential equations and integral equations as models, introduction to simulation techniques. Ordinary Differential Equations:Modeling with first order differential equations, Newton's law of cooling, radioactive decay,motion in a gravitational field, population growth, mixing problem, Newtonian mechanics,modeling with second order differential equations, vibrations, application to biological systems, modeling with period or impulse forcing functions,modeling with systems of first order differential equations, competitive hunter model, predator prey model. Partial Differential Equations: Methodology of mathematical modeling, objective, background, approximation and idealization, model validation, compounding, modeling wave phenomena (wave equation), shallow water waves, uniform transmission line, traffic flow, RC circuits,modeling the heat equation and some application to heat conduction problems in rods, lamina, cylinders etc, modeling the potential equation (Laplace equation), applications in fluid mechanics, gravitational problems,equation of continuity.

Recommended Books:

1. Mooney, D. D., & Swift, R. J. (2021). *A course in mathematical modeling* (Vol. 13). American Mathematical Society.
2. Makinia, J., & Zaborowska, E. (2020). *Mathematical modelling and computer simulation of activated sludge systems*. IWA publishing.
3. Chidambaram, M. (2018). *Mathematical Modelling and Simulation in Chemical Engineering*. Cambridge University Press.
4. de Castro, A. B., Gómez, D., & Salgado, P. (2014). *Mathematical models and numerical simulation in electromagnetism* (Vol. 74). Springer.
5. Neittaanmäki, P., Repin, S., & Tuovinen, T. (Eds.). (2016). *Mathematical modeling and optimization of complex structures*. New York: Springer.
6. Brauer, F., Castillo-Chavez, C., & Feng, Z. (2019). *Mathematical models in epidemiology* (Vol. 32). New York: Springer.
7. Palagin, A., Anisimov, A., Morozov, A., & Shkarlet, S. (2020). *Mathematical Modeling and Simulation of Systems*. Springer International Publishing.

Course Name: Vector Analysis	**Course Code: MTH-645**
Course Structure: Lectures: 3, Lab:0	**Credit Hours:** 3
Prerequisites: Calculus-II	

Specific Objectives of course:

- This course, focuses on the physical meaning of vector analysis formulae by learning practical applications as well as the mathematical proofs of the formulae.
- This course offers students both an opportunity of systematic learning of vector and its applications and an introduction to advanced courses such as continuum mechanics, fluid dynamics, and electromagnetics.

Course Outline:

3-D vectors, summation convention, kronecker delta, Levi-Civita symbol, vectors as quantities transforming under rotations with ϵ_{ijk}notation, scalar and vector-triple products, scalar and vector-point functions, differentiation and integration of vectors, line integrals, path independence, surface integrals, volume integrals, gradient, divergence and curl with physical significance and applications, vector identities, Green's theorem in a plane, divergence theorem, Stokes' theorem, coordinate systems and their bases, the spherical-polar and the cylindrical-coordinate.

Learning Outcomes:

At the end of this course, students will be able to:

- Perform the basic algebra operation of vectors.
- Understand the differential operation of vectors and apply it to the calculation of particle motion and rotation of body.
- Understand the gauging of curved line and surface with vectors, and calculate the values.
- Understand the gradient of scalar field and the divergence and rotation of vector field, and calculate those values.

Solve practical problems using the integral theorems of vector.

Recommended Books:

1. MD. Ashraf Ali, Vector Analysis, 4th Edition, New Age Int, Publisher, 2018
2. C. E. B. 1884 Weatherburn, Elementary Vector Analysis, With Application to Geometry and Physics, Creative Media Partners, LLC Publisher, 2018
3. Alexander MacFarlane, Vector analysis and Quaternions, 1st Edition, Independently

Published, 2019

4. Eutiquio C. Young, Vector and Tensor Analysis, 2nd Edition, CRC Press, 2019
5. John Vince, Vector Analysis for Computer Graphics, 2nd Edition, Springer London Publisher, 2021
6. Josiah Willard Gibbs, Elements of Vector Analysis,1st Edtion, LEGARE STREET Press Publisher, 2023

ANNEXTURE-B

INTERDISIPLINARY/ALLIED COURSES

Course Name: Calculus and Analytical Geometry	**Course Code: MTH-301**
Course Structure: Lectures: 3, Labs: 0	**Credit Hours:** 3
Prerequisites: Calculus	
Course Objectives: To provide foundation and basic ground for calculus and analytical geometry background. **Course Outline:** Functions and graphs, limits and continuity, differential calculus; (concept and idea of differentiation geometrical and physical meaning of derivative, rules for differentiation, techniques of differentiation, chain rule, implicit differentiation, linear approximation, applications of differentiation, extrema of functions, mean value theorem, maximum and minima, concavity , integral calculus; concept and idea of integration, indefinite, techniques of integration, Riemann sums and definite integrals, applications of integrals, physical interpretation as areas, mean value theorem, areas between curves, finding volumes by slicing, volumes and surface of revolution, Geometry in Two Dimensions: Cartesian-coördinate mesh, slope of a line, equation of a line, parallel and perpendicular lines, various forms of equation of a line, intersection of two lines, angle between two lines, distance between two points, distance between a point and a line. Circle; Equation of a circle, circles determined by various conditions, intersection of lines and circles, locus of a point in various conditions. Conic Sections; Parabola,	

ellipse, hyperbola. **Intended Learning Outcomes:** At the end of the course student would be able: • to solve problem using calculus concepts of limits, derivations, integration analytically and graphically, use calculus concepts in computer applications.
Reference Material: 1. Edwars, C.H., Penney, D.E.,.(2002).Calculus.Prentice hall,Inc.. 2. Anton, H.., Bivens, I., Devis, S. (7th edition).(2002). Calculus. Newyork: John Wiley and sons, Inc. 3. Thomas, G.B.(1992).Calculus and analytical geometry .Addison Welsey Longman, Inc.

Course Name: Advanced Calculus	**Course Code: MTH-302**
Course Structure: Lectures: 3, Labs: 0	**Credit Hours: 03**
Prerequisites: Calculus	
Course Objective 1. This course will familiarize students with the basic principles of calculus and their application to problem solving. 2. To enable the students about practical applications of the course in different fields. **3.** Make the students to polish their analytical skills.	
Course Outline: Introduction to Limits: Theorems of limits and their application to functions: Introduction to Continuity,. Derivatives: Introduction to derivatives, Partial derivatives and their geometrical significance Application problems (rate of change, marginal analysis) Higher derivatives: Mean value theorem, Applications of derivatives: Curvature and radius of curvature, maxima and minima of a function. Application partial derivatives: Integral calculus: Vector differentiation, vector integration and their application. Laplace transforms, Fourier series, Z-Transform.	

Learning Outcomes:

- Eplain the basic properties of the real number system;
- apply theorems of analysis to real functions of one variable and prove basic analysis results
- understand the fundamentals of mathematical analysis through a rigorous study of the calculus of real functions of one variable ;and.

Recommended Books:

1. Calculus by Thomas Finney, 11th Edition, Dec 26, 2010.

2. Brief Calculus and its applications by Doniel D.Benice, 5th Edition, 1997.

3. Applied Calculus by Raymond A. Barnett, 5th Edition, 08/28/1996

4. Calculus by Gerald L. Bradley, 2nd Edition, 2002.

Course Name: Multivariate Calculus	**Course Code: MTH-404**
Course Structure: Lectures: 3, Labs: 0	**Credit Hours:** 3
Prerequisites: Calculus and Analytical Geometry	

Course Objectives:

The goals are to develop the skills to have ground knowledge of multivariate calculus and appreciation for their further computer science courses.

Course Outline:

Functions of Several Variables and Partial Differentiation. Multiple Integrals, Line and Surface Integrals. Green's and Stoke's Theorem. Fourier Series: periodic functions, Functions of any period P-2L, Even & odd functions, Half Range expansions, Fourier Transform. Laplace Transform, Z-Transform.

Intended Learning Outcomes:

Students will be able: to apply mathematical and computational methods to range of application problems in multivariate calculus, evaluate partial derivations and multiple integrals of multivariate functions.

Reference Material:

1. Stewart,J. (6^{th} ed.).(2007).*Multivariable Calculus*. Cengage Learning publishers.
2. *Swokowski,Olinick,M.,Pence,D.(6^{th} ed.). (1994). Calculus and Analytical Geometry. Thomson Learning EMEA, Ltd.*
3. Anton,H ,Herr,A.(5^{th} Ed.).(1995). *Multivariable Calculus*. John Wiley.

Course Title: Linear Algebra	**Course Code: MTH-405**
Course Structure: Lectures: 3	**Credit Hours:** 3
Prerequisites: Calculus	

Course Objective: To provide fundamentals of solutions for systems of linear equations, operations on a system of equations, matrix properties, solutions, and study of their properties, to enable the students about Practical applications in Bio-Informatics, the aim is to provide a practical description of the topics, tools, issues and current trends in the fields including their impact on biology and human health and medicine. Make students polish their analytical skills. Have well understanding to utilize this course in this program.

Course Outline: Systems of Linear Equations and Matrices; Introduction to systems of linear equations, Gaussian Elimination, Matrices and Matrices Operations, Inverses, Rules of Matrix Arithmetic, Elementary Matrices and a method for finding inverse, systems of equations and invertibility, diagonal triangular and symmetric matrices, Determinants; the determinant , evaluating determinants by row reduction, properties of the determinant function, co-factor, crammer’s rule, Vector in 2space and 3 space, introduction of vector, norm, vector athematic , dot product cross product, General vector spaces; real vector spaces, subspace, Linear Independence, basis and dimension, row space ,column space and null space, rank and nullity , Eigen values and Eigen vector, diagonalization, Orthogonal diagonalization, Linear transformation; general linear transformation, kernel and range, matrices of general linear transformation.

Intended Learning Outcomes: Students will understand: some applications of the system of linear equations, and apply matrix multiplications in digraphs and communication matrices.

Recommended Books:

1. Anton, H., (2018). Elementary Linear Algebra, Eighth Edition, United Kingdom: Wiley.
2. Anton, H., Rorres, C., Kaul, A. (2019). Elementary Linear Algebra. United Kingdom: Wiley.
3. Messer, R. (2021). Linear Algebra: Gateway to Mathematics. United States: American

Mathematical Society.
4. Boyd, S., Vandenberghe, L. (2018). Introduction to Applied Linear Algebra: Vectors, Matrices, and Least Squares. United Kingdom: Cambridge University Press.

Course Name: Differential Equations	**Course Code: MTH-501**
Course Structure: Lectures: 3, Labs: 0	**Credit Hours:** 3
Prerequisites: Calculus and Analytical Geometry	

Course Objectives:

Develop fundamental skills of solving ordinary differential equations, and developing differential equations for real-world problems

Course Outline:

Ordinary Differential Equations of the First Order: Geometrical Considerations, Isoclines, Separable Equations, Equations Reducible to Separable Form, Exact Differential Equations, Integrating Factors, Linear First-Order Differential Equations, Variation of Parameters. Ordinary Linear Differential Equations; Homogeneous Linear Equations of the Second Order, Homogeneous Second-Order Equations with Constant Coefficients, General Solution, Real Roots, Complex Roots, Double Root of the Characteristic Equation, Differential Operators, Cauchy Equation, Homogeneous Linear Equations of Arbitrary Order, Homogeneous Linear Equations of Arbitrary Order with Constant Coefficients, Non-homogeneous Linear Equations. Modeling of Electrical Circuits. Systems of Differential Equations. Series Solutions of Differential Equations.

Intended Learning Outcomes:

Students will be able to solve 1^{st} and 2^{nd} order ODES, able to apply ODES in computer applications, solve ODES such as Laplace, heat and wave equations using separation of variables.

Reference Material

1. Greenberg,M.D.(*1996).Advanced Engineering Mathematics.* Prentice Hall publishers.

2. Kreyszig,E. (7th ed.).(1993). *Advanced Engineering Mathematics*. John Wiley & Sons Inc.
3. Zill,D.G.,Prindle,Weber,Schmidt.(1996).*A First Course in Differential Equation.* Brooks/Cole Publishing,
4. Zill,D.G., Cullen,M.R. (1996). *Differential Equations with Boundary-Value Problems*, , Brooks/Cole Publishing,
5. Edwards,C.H., Penney., David, E. (1993). Penney. *Elementary Differential Equations With Applications*, Prentice Hall.

Course Name: Computational Mathematics	**Course Code: MTH-502**
Course Structure: Lectures: 3, Labs: 0	**Credit Hours: 03**
Prerequisites: None	

Course Objective

1. Students will work with various sorting and searching techniques.
2. Have will understanding to utilize this course in this program.
3. Make students to polish their computational skills.

Course Outline:

Mathematical Preliminaries: Introduction Solution of equation in one variable. **Operators** Introduction. Types of operators. Delta, Nebla. Sigma, average. Shift. Relation of operators. Proofs of operators. Newton's forward difference formula Derivation. Construction of table for Delta. **Interpolation (equally spaced data)** Newton's backward difference formula derivation. Difference table for Nebla. **Sterling's interpolation formula, Derivation. Interpolation (unequally spaced dated).** Newton's divided difference formula for unequally spaced data. Derivation. Lagrange's formula for interpolation. Derivation. **Solution of system of linear equation.** Definition. Jacobi iterative method. Gauss sidle iterative method. SOR method. Solution of Initial value problems for ODE's by Euler's method, Taylor's method and Runge-Kutta method. **Iterative techniques in the numerical solution of system of non-linear equations.** Bisection method. The Newton Rap son method. The Secant method and Regulara Falsi method**.** Fixed point Iteration method. Least square approximation. Chebyshev approximation Pade's approximation. **Eigen vales and Eigen vectors.** Definition and Properties of Eigen vales and Eigen vectors Types. The power method. Exercise. Inverse power method. Applications of Eigen vales and Eigen vectors.

Learning Outcomes:

- Understand the theoretical and practical aspects of the use of numerical analysis.

- proficient in implementing numerical methods for a variety of multidisciplinary applications.
- establish the limitations, advantages, and disadvantages of numerical analysis.
- derive numerical methods for various mathematical operations and tasks, such as interpolation, differentiation, integration, the solution of linear and nonlinear equations, and the solution of differential equations.

Recommended Books:

1. Richard L. Burden, J. Douglas Faires, Numerical Analysis, 2005.
2. Josef Stoer, Roland Bulirsch, Introduction to Numerical analysis, 2002.
3. Richard L. Burden, J. Douglas Faires, Numerical Methods.
4. An Introduction to Numerical Analysis (Text book for M.Sc.)

Course Name: Geometry & Drafting	**Course Code: MTH-303**
Course Structure: Lectures:1, Practical: 2	**Credit Hours:** 3
Prerequisites: None	

Course Objectives:

- Familiarization/Recognition of Student with drafting and Geometry (Measuring Systems).
- To train students in acquiring all the drafting skills by using tools (Matrices).
- To understand different methodologies of constructing various basic polygons Including reducing /enlargement of figures.
- One point perspective, two point perspective and different views.

Course Content:

ntroduction to basic calculations and measurements. Introduction of Geometry(what is geometry ,use of geometry tools in precise manners) Construction of square (by using campus) Division of Line (division line without measurements.) Angles and ellipse (construction of angles with campus/construction of ellipse). Construction of polygons. Introduction of drafting (scale and proportions). One point perspectives (drawing of interior on scale). Two points perspectives (Drawing of Exterior on Scale) Exam Projects.

Learning Outcomes:

After studying this course students will be able to

- Understand and recognize drafting and Geometry (Measuring Systems).

- Acquire all the drafting skills by using tools (Matrices).
- Understand different methodologies of constructing various basic polygons Including reducing /enlargement of figures. One point perspective, two point perspective and different views.

Recommended Books/Helping Material:

Book: Geometrical Drawing for art students by I. H. MORRIS
Material: T-square, set –square, campus, clutch pencils, drafting board etc.

Course Name: Business Mathematics	**Course Code: MTH-304**
Course Structure: Lectures: 3	**Credit Hours:** 3

Prerequisites: None

Course Objectives

This course is built upon the mathematical concepts, principles and techniques that are useful in business management. The main objectives of the course are to enhance students' competency in application of mathematical concepts in solving business management problems and to improve their level of quantitative approach.

Course Contents:

Mathematical Notations. Quadratic Equations. Linear Equations. Systems Of Linear Equations And Their Applications. Nonlinear Functions And Their Applications
Matrices Determinants

Indented Learning Outcomes

Upon the successful completion of this course, you should be able to:

- Mathematical Function
- Building and solving linear and quadratic equations
- Types of function
- Matrices and its applications
- Determinants and its applications

Recommended Books

1. Cheryl Cleaves, Business Maths, Pearson (Latest Edition)
2. Burton, Shelton, Business Maths using Excel, South-Western Cengage Learning
3. Budnick, Mathematics for Business Economics and Social Science

ANNEXTURE-C

ALIGNED COURSES OF MPHIL/PHD PROGRAM

SESSION(2023-ONWARDS)

Cours Code	Title	Credit Hours.
MTH-701	Advanced measure theory	3
MTH-702	Advanced topology	3
MTH-703	Advanced functional analysis	3
MTH-704	Astronomy	3
MTH-705	Astrophysics	3
MTH-706	Advanced numerical analysis	3
MTH-707	Advanced fluid dynamics	3
MTH-708	Advanced mathematical methods	3
MTH-709	Advanced approximation theory	3
MTH-710	Advanced operation research	3
MTH-711	Applied functional Analysis	3
MTH-712	Advanced partial differential equations	3
MTH-713	Advanced mathematical statistics	3
MTH-714	Analytical dynamics-I	3
MTH-715	Analytical dynamics-II	3
MTH-716	Advanced analytical dynamics-I	3
MTH-717	Advanced analytical dynamics-II	3
MTH-718	Basics theory of the fluids	3

MTH-719	Banach algebra	3
MTH-720	Computational fluid dynamics	3
MTH-721	Cosmology	3
MTH-722	Classical theory of fields	3
MTH-723	Computer vision (variational)	3
MTH-724	Differential geometry	3
MTH-725	Digital image processing(variational)	3
MTH-726	Elastodynamics	3
MTH-727	Electrodynamics-I	3
MTH-728	Electrodynamics-II	3
MTH-729	Fixed point theory	3
MTH-730	Field theory	3
MTH-731	Gas dynamics	3
MTH-732	Multigrid method for linear and non-linear PDEs	3
MTH-733	Mathematical Modeling and Numerical Simulation	3
MTH-734	Mathematical techniques for boundary value problems	3
MTH-735	Magnetohydrodynamics-I	3
MTH-736	Magnetohydrodynamics-II	3
MTH-737	Mathematical biology	3
MTH-738	Numerical Solutions of Ordinary Differential Equations	3
MTH-739	Numerical Solutions of PDE’s	3
MTH-740	Numerical Solutions for Integral Equations	3
MTH-741	Numerical linear algebra	3

MTH-742	Non-Newtonian fluid mechanics	3
MTH-743	Parallel algorithms	3
MTH-744	Plasma theory-I	3
MTH-745	Plasma theory-II	3
MTH-746	Perturbation methods-I	3
MTH-747	Perturbation methods-II	3
MTH-748	Rings and Categories of Modules	3
MTH-749	Riemann surfaces	3
MTH-750	Research methodology	3
MTH-751	Regression analysis	3
MTH-752	Spectral Theory in Hilbert Spaces	3
MTH-753	Statistical decision theory	3
MTH-754	Several complex variables	3
MTH-755	Topological vector spaces	3
MTH-756	Viscous fluid-I	3
MTH-757	Viscous fluid-II	3
MTH-758	Fractional Calculus	3
MTH-759	Fractional Differential Equations	3
MTH-760	Fuzzy Fixed Point Theory	3
MTH-761	Fuzzy Sets and Their Applications	3
MTH-801	Advanced group theory	3
MTH-802	Special functions	3
MTH-803	Advanced special functions	3
MTH-804	Advanced integral equations	3

MTH-805	Nonlinear dynamics and chaos	3
MTH-806	Advanced general relativity	3
MTH-807	Applied data analysis techniques	3
MTH-808	Advanced mathematical physics	3
MTH-809	Advanced mathematical modeling	3
MTH-810	Nonlinear optimization	3
MTH-811	Unconstrained optimization	3

Course Name: Advanced Group Theory	**Course Code: MTH-701**
Course Structure: Lectures: 3	**Credit Hours:** 3

Survey of group theory, (elementary concepts) symmetric and alternating groups of finite degree, orbit of symmetric and alternating groups, transitive group. Group action on sets, Group action on groups, periodic torsion free and mixed abelian groups, free abelian groups. Finitely generated groups, Definition of a modular lattices and distributive lattices, the lattices of all sub groups of a group, the lattice of admissible, sub group, the lattice of normal subgroups.

Recommended Books

- Roman, S. (2011). *Fundamentals of group theory: an advanced approach*. Springer Science & Business Media.
- Hall, M. (2018). *The theory of groups*. Courier Dover Publications.
- Hamermesh, M. (2012). *Group theory and its application to physical problems*. Courier Corporation.
- Knapp, A. W., & Knapp, A. W. (1996). *Lie groups beyond an introduction* (Vol. 140). Boston:

Birkhäuser.

Course Name: Advanced Measure Theory	Course Code: MTH-702
Course Structure: Lectures: 3	**Credit Hours:** 3

Interated integration, Fubini's theorem, Tonelli Hobson Theorem, Vitali's cover of sets. Vitli's Lemma, Lipschutz conditions. Relation between Stieltjes and Lebesgue integral, Riesz Fischer Theorem. Measure spaces, signed measure, Hahn decomposition, Hahn decomposition theorem, Jordan decomposition, singular measures, Jordan decomposition theorem, Random Nikodym theorem, Random Nikodym derivative. Lebesgue decomposition theorem, Product measure.

Recommended Books:

- Evans, L. (2018). *Measure theory and fine properties of functions*. Routledge.
- Halmos, P. R. (2013). *Measure theory* (Vol. 18). Springer.
- Shultz, K. S., Whitney, D., & Zickar, M. J. (2020). *Measurement theory in action: Case studies and exercises*. Routledge.
- Gordon, R. A. (1994). *The integrals of lebesgue, denjoy, perron, and henstock* (No. 4). American Mathematical Soc..

Course Name: Advanced Topology	Course Code: MTH-703
Course Structure: Lectures: 3	**Credit Hours:** 3

Course Outline:

Product Topology, Point-Open Topology, Compact-open topology, Weak Topology-1, Weak Topology-2, Quotient Topology, Identifications, Identification maps, Fundamental Group(first homotopy group), Elementary properties of the fundamental group, Countinuous functions and homomorphisms, Categories and functors, Seifert-Van Kampan Theorem, Direct limits, lifting theorems, regular covering spaces, map liftings, universal covering spaces. 1-manifold on contractability of S1, Jordan Curve theorem.

Recommended Books:

- Montgomery, D., & Zippin, L. (2018). *Topological transformation groups*. Courier Dover Publications.
- Singh, T. B. (2019). *Introduction to topology*. Springer.
- Naimpally, S. A., & Peters, J. F. (2013). *Topology with applications: topological spaces via near and far*. World Scientific.
- Nakahara, M. (2018). *Geometry, topology and physics*. CRC press.

Course Name: Advanced Functional Analysis	**Course Code: MTH-704**
Course Structure: Lectures: 3	**Credit Hours:** 3

Course Outline:

Metric Sapces

The definition and some examples, convergence, completeness and Baire's Theorem. Continuous mapping, spaces of continuous functions.

Banach Spaces

The definition and some examples, Continuous linear transformations. Linear operations on a normed linear space. Continuous linear functional. The Hahn Banach theorem. The open mapping Theorem, closed graph Theorem, the conjugate of an operator.

Hilbert spaces

The definition and some properties, normed vector spaces, orthogonality, complete orthogonal sets, the conjugate space H, Inner product in H, the adjoint of an operator, self adjoint operators, normal and Unitary operators.

Banach Algebras and Spectral Theory

Introduction, complex homomorphisms, basic properties of spectra, symbolic calculus, differentiation.

Recommended Books:

- Edwards, R. E. (2012). *Functional analysis: theory and applications*. Courier Corporation.
- *R.Beattie, Convergence Structures and Applications to Functional Analysis, Kluwer Academic Publishers,2007.*
- Kreyszig, E. (1991). *Introductory functional analysis with applications* (Vol. 17). John Wiley & Sons.
- Kutateladze, S. S. (2013). *Fundamentals of functional analysis* (Vol. 12). Springer Science & Business Media.
- Pedersen, M. (2018). *Functional analysis in applied mathematics and engineering*. CRC press.
- Lebedev, L. P., Vorovich, I. I., & Gladwell, G. M. L. (2012). *Functional analysis: applications in*

mechanics and inverse problems (Vol. 41). Springer Science & Business Media.

Course Name: Astronomy	**Course Code: MTH-705**
Course Structure: Lectures: 3	**Credit Hours:** 3

Course Outline:

Spherical trigonometry. The Calestial sphere. Contents of the Universe. Tools of the astronomer. Refraction. The meridian circle. The Hertzsprung-Russell diagram. Stellar evolutions. Planetary motions. Time. Planetary phenonmenon and Heliographic co-ordinates. Aberration. Parallax. Galaxies and their evolution, Cosmolgy.

The solar system. Earth, Moon and appollo findings. Venus, Mars, Jupiter and Saturn. Precession and Nutation. The Proper motions of the stars. Astronomical photography. Binary star orbits. Occulations and eclipses. Life in the cosmos. Cosmic evolution.

Recommended books:

- Erot, A., & Clarke, D. (2000). *Astronomy Principles & Practice*.
- Trumpler, R. J., & Weaver, H. F. (2023). *Statistical astronomy*. Univ of California Press.
- Kutner, M. L. (2003). *Astronomy: A physical perspective*. cambridge university press.
- Filipović, M. D., & Tothill, N. F. (2021). *Principles of Multimessenger Astronomy*. IOP Publishing.
- Lauterbach, T. (2022). *Radio Astronomy: Small Radio Telescopes: Basics, Technology, and Observations*. Springer Nature.

Course Name: Advanced Measure Theory	**Course Code: MTH-706**
Course Structure: Lectures: 3	**Credit Hours:** 3

Course Outline:

Static stellar structure and the equilibrium conditions. Introduction to stellar modelling. The Hertzprung-Russell diagram and stellar evolution. Gravitational collapse and degenerate stars. White dwarfs, neutron stars and black holes. Systems of stars, irregular and globular clusters, galaxies superclusters and filaments. Astrophysical dark matter and galactic haloes.

Recommended Books:

- Carroll, B. W., & Ostlie, D. A. (2017). *An introduction to modern astrophysics*. Cambridge University Press.
- Lang, K. (2006). *Astrophysical Formulae: Volume I & Volume II: Radiation, Gas Processes and High Energy Astrophysics/Space, Time, Matter and Cosmology*. Springer Science & Business Media.
- Padmanabhan, T. (2006). *An invitation to astrophysics* (Vol. 8). World Scientific.
- Padmanabhan, T. (2000). *Theoretical astrophysics: volume 3, galaxies and cosmology* (Vol. 3). Cambridge University Press.
- Gordon, R. A. (1994). *The integrals of lebesgue, denjoy, perron, and henstock* (No. 4). American Mathematical Soc..

Course Name: Numerical Analysis	**Course Code: MTH-707**
Course Structure: Lectures: 3	**Credit Hours:** 3

Course Outline:

Review of the basic concepts in numerical analysis with convergence and error estimate. Initial value problem, Euler method, multistep methods and their consistency, stability criteria and convergence, system of differential equations, boundary value problems, finite difference methods, collection method, Spline methods, Rayleigh-Ritz methods, Galerikin methods.

Recommended Books:

- Chakraverty, S., Mahato, N., Karunakar, P., & Rao, T. D. (2019). *Advanced numerical and semi-analytical methods for differential equations*. John Wiley & Sons.
- John, F. (1990). *Nonlinear wave equations, formation of singularities* (Vol. 2). American Mathematical Soc..
- Linz, P. (2019). *Theoretical numerical analysis*. Courier Dover Publications.
- Eisley, J. G., & Waas, A. M. (2011). *Analysis of structures: an introduction including numerical methods*. John Wiley & Sons.
- Butcher, J. C. (2016). *Numerical methods for ordinary differential equations*. John Wiley & Sons.

Course Name: **Advanced Fluid Dynamics**	**Course Code: MTH-708**
Course Structure: Lectures: 3	**Credit Hours:** 3

Course Outline:

Review of gradient, divergence and curl. Elementary idea of tensors. Velocity of fluid, Streamlines and path lines, Steady and unsteady flows, Velocity potential, Vorticity vector, Conservation of mass, Equation of continuity. Equations of motion of a fluid, Pressure at a point in fluid at rest, Pressure at a point in a moving fluid, Euler's equation of motion, Bernoulli's equation. Singularities of flow, Source, Sink, Doublets, Rectilinear vortices. Complex variable method for two-dimensional problems, Complex potentials for various singularities, Circle theorem, Blasius theorem, Theory of images and its applications to various singularities. Three-dimensional flow, Irrotational motion, Weiss's theorem and its applications. Viscous flow, Vorticity dynamics, Vorticity equation, Reynolds number, Stress and strain analysis, Navier-Stokes equation, Boundary layer Equations.

Recommended Books:

- Rieutord, M. (2014). *Fluid dynamics: an introduction*. Springer.
- Chorlton, F. (2016). *Textbook of fluid dynamics*. CBS Publishers & Distributors Pvt Ltd.
- Paterson, A. R. (1983). *A first course in fluid dynamics*. Cambridge university press.
- Kleinstreuer, C. (2018). *Modern fluid dynamics*. Boca Raton, FL, USA: CRC Press.

Course Name: **Advanced Mathematical Methods**	**Course Code: MTH-709**
Course Structure: Lectures: 3	**Credit Hours:** 3

Course Outline:

General solution of Bessel equation, Recurrence relations, Orthogonal sets of Bessel functions, Modified Bessel functions, Applications. General solution of Legendre equation, Legendre polynomials, Associated Legendre polynomials, Rodrigues formula, Orthogonality of Legendre polynomials, Application. Concept and calculation of Green’s function, Approximate Green’s function, Green’s function method for differential equations, Fourier Series, Generalized Fourier series, Fourier Cosine series, Fourier Sine series, Fourier integrals. Fourier transform, Laplace transform, Z-transform, Hankel transform, Mellin transform. Solution of differential equation by Laplace and Fourier transform methods.

Recommended Books:

- Bowman, F. (2012). *Introduction to Bessel functions*. Courier Corporation.
- Kythe, P. K., Schäferkotter, M. R., & Puri, P. (2018). *Partial differential equations and Mathematica*. Chapman and Hall/CRC.
- Poularikas, A. D., & Grigoryan, A. M. (2018). *Transforms and applications handbook*. CRC press.
- Di Nunno, J., & Oksendal, B. (Eds.). (2011). *Advanced mathematical methods for finance*. Springer Science & Business Media.
- Hayek, S. I. (2000). *Advanced mathematical methods in science and engineering*. CRC Press.
- Richards, D. (2002). *Advanced mathematical methods with Maple*. Cambridge University Press.

Course Name: **Advanced Approximation Theory**	**Course Code: MTH-710**
Course Structure: Lectures: 3	**Credit Hours:** 3

Course Outline:

The approximation problem and existence of best approximations. The uniqueness of best approximations. Approximation operators. Polynomial interpolation. Uniform convergence of

polynomial approximations. Least squares approximations. Properties of orthogonal polynomials, order of convergence of polynomial approximations. Interpolation by piecewise polynomials. Chebyshev polynomials.

Recommended Books:

- Anastassiou, G. A., & Gal, S. G. (2012). *Approximation theory: moduli of continuity and global smoothness preservation*. Springer Science & Business Media.
- Achieser, N. I. (2013). *Theory of approximation*. Courier Corporation.
- Phillips, G. M. (2003). *Interpolation and approximation by polynomials* (Vol. 14). Springer Science & Business Media.
- Trefethen, L. N. (2019). *Approximation Theory and Approximation Practice, Extended Edition*. Society for Industrial and Applied Mathematics.

Course Name: **Applied Functional Amalysis**	**Course Code: MTH-711**
Course Structure: Lectures: 3	**Credit Hours:** 3

Course Outline:

Applications to bounded linear functional, Application to submmability of sequences,

Numerical Integration and weak* convergence, Banach fixed point theorem and its applications to linear equations, differential equations and integral equations, Unbounded linear operators in quantum mechanics.
Recommended Books: Papageorgiou, N. S., & Winkert, P. (2018). *Applied Nonlinear Functional Analysis: An Introduction*. Walter de Gruyter GmbH & Co KG. Zeidler, E. (2012). *Applied functional analysis: applications to mathematical physics* (Vol. 108). Springer Science & Business Media. Oden, J. T., & Demkowicz, L. (2017). *Applied functional analysis*. CRC press. Pedersen, M. (2018). *Functional analysis in applied mathematics and engineering*. CRC press. Dieudonné, J. (2020). History of Functional Analyais. In *Functional Analysis, Holomorphy, and Approximation Theory* (pp. 119-129). CRC Press.

Course Name: Advanced Partial Differential Equations	**Course Code: MTH-712**
Course Structure: Lectures: 3	**Credit Hours:** 3
Course Outline: PDEs with at least 3 independent variables; non-homogenous problems; green function for time independent problems, infinite domain problems; green function of time dependent problems, wave equation and the method of characteristicsGarabedian, P. R. (2023). *Partial differential equations* (Vol. 325). American Mathematical Society.	
Recommended Books: Zang, Y., Bao, G., Ye, X., & Zhou, H. (2020). Weak adversarial networks for high-dimensional partial differential equations. *Journal of Computational Physics*, *411*, 109409. Salsa, S., & Verzini, G. (2022). *Partial differential equations in action: from modelling to theory* (Vol. 147). Springer Nature. Blechschmidt, J., & Ernst, O. G. (2021). Three ways to solve partial differential equations with neural	

Course Name: Advanced Mathematical Statistics	**Course Code: MTH-713**

Course Structure: Lectures: 3	**Credit Hours:** 3

Course Outline:

Univariate probabilistic and deterministic models, Methods of estimation, Composition of confidence intervals and testing, Optimal tests and confidence intervals, Likelihood ratio tests, Linear models, Regression and correlation, Analysis of variance, Analysis of discrete data, Non-parametric models, Decision theory, Markov processes.

equations and integral equations, Unbounded linear operators in quantum mechanics.

Recommended Books:

Murphy, K. P. (2023). *Probabilistic machine learning: Advanced topics*. MIT press.

Kliestik, T., Valaskova, K., Nica, E., Kovacova, M., & Lazaroiu, G. (2020). Advanced methods of earnings management: Monotonic trends and change-points under spotlight in the Visegrad countries. *Oeconomia Copernicana*, *11*(2), 371-400.

Manoukian, E. B. (2022). *Mathematical nonparametric statistics*. Taylor & Francis.

Mertler, C. A., Vannatta, R. A., & LaVenia, K. N. (2021). *Advanced and multivariate statistical methods: Practical application and interpretation*. Routledge.

Ramachandran, K. M., & Tsokos, C. P. (2020). *Mathematical statistics with applications in R*. Academic Press.

Course Name: Analytical Dynamics-I	**Course Code: MTH-714**
Course Structure: Lectures: 3	**Credit Hours:** 3

Course Outline:

Equations of dynamics and its various forms, Equations of Lagrange and Euler, Jacobi's elliptic functions and the qualitative and quantitative Solutions of the problems of Euler and Poisson, The Problems of Lagrange and Poisson, Dynamical system, Equations of Hamilton and Appell, Hamilton–Jacobi theorem; Separable systems' Holder's variational principles and its consequences.

Recommended Books:

Fang, Y., Nie, Y., & Penny, M. (2020). Transmission dynamics of the COVID- 19 outbreak and effectiveness of government interventions: A data- driven analysis. *Journal of medical virology*, *92*(6), 645-659.

Staniszkis, J. (2023). *The dynamics of the breakthrough in Eastern Europe: the Polish experience* (Vol. 6). Univ of California Press.

Shabana, A. A. (2020). *Dynamics of multibody systems*. Cambridge university press.

Zhai, W., & Zhai, W. (2020). *Vehicle–track coupled dynamics models* (pp. 17-149). Springer Singapore.

Tu, J., Yeoh, G. H., Liu, C., & Tao, Y. (2023). *Computational fluid dynamics: a practical approach*. Elsevier.

Course Name: Analytical Dynamics-II	**Course Code: MTH-715**
Course Structure: Lectures: 3	**Credit Hours:** 3

Course Outline:

Groups of continuous Transformations and Poincare's Equations, Systems with one degree of freedom; Singular Points, Cycle characteristics of systems with a Degree of freedom, Ergodic theorem, Matric indecompossability, stability of motion periodic Orbits.

Recommended Books:

De Doncker, R. W., Pulle, D. W., & Veltman, A. (2020). *Advanced electrical drives: analysis, modeling, control*. Springer Nature.

Tu, J., Yeoh, G. H., Liu, C., & Tao, Y. (2023). *Computational fluid dynamics: a practical approach*. Elsevier.

Zhai, W., & Zhai, W. (2020). *Vehicle–track coupled dynamics models* (pp. 17-149). Springer Singapore.

Course Name: Advanced Analytical Dynamics-I	**Course Code: MTH-716**
Course Structure: Lectures: 3	**Credit Hours:** 3

Course Outline:

Equations of dynamic and its various forms. Equations of Langrange and Euler. Jacobi's elliptic functions and the qualitative and quantitative solutions of the problem of Euler and Poisson. The Problems of Langrange and Poisson. Dynamical system. Equations of Hamilton and Appell. Hamilton-Jacobi theorem. Separable systems. Holder's variational principle and its consequences.

Recommended Books:

Richardson, K. D. (2022). *Advanced Isogeometric Discretization Techniques* (Doctoral dissertation, Brigham Young University).

Özsoy, E. (2020). *Geophysical Fluid Dynamics I*. Springer.

Ducrot, A., Griette, Q., Liu, Z., & Magal, P. (2022). *Differential Equations and Population Dynamics I*. Springer, Cham.

Qudrat-Ullah, H., & Asif, M. (2020). *Dynamics of Energy, Environment and Economy*. Springer International Publishing.

Course Name: **Advanced Analytical Dynamics-II**	**Course Code: MTH-717**
Course Structure: Lectures: 3	**Credit Hours:** 3

Course Outline:

Groups of continuous transformations and Poincare's equations. Systems with one degree of freedom, Singular points, Cyclic characteristics of systems with a degree of freedom. Ergodic theorem, Metric indecompossability. Stability of motion.

Recommended Books:

- Mannouch, J. R., & Richardson, J. O. (2020). A partially linearized spin-mapping approach for nonadiabatic dynamics. II. Analysis and comparison with related approaches. *The Journal of Chemical Physics*, *153*(19).
- Pathak, H., Sato, T., & Ishikawa, K. L. (2020). Time-dependent optimized coupled-cluster method for multielectron dynamics. II. A coupled electron-pair approximation. *The Journal of chemical physics*, *152*(12).
- Velasquez-Martinez, L. F., Zapata-Castano, F., & Castellanos-Dominguez, G. (2020).

Dynamic Modeling of Common Brain Neural Activity in Motor Imagery Tasks. *Frontiers in Neuroscience*, *14*, 714.

Course Name: Basic Theory of Fluids	**Course Code: MTH-718**
Course Structure: Lectures: 3	**Credit Hours:** 3

Course Outline:

Euler's equation of motion, Viscosity, Navier-Stoke's equation and exact solutions, Dynamical similarity and Reynold's number, Turbulent flow, Boundary layer concept and governing equations, Reynold's equations of turbulent motion. Magnetohydrodynamics, MHD equations, Fluid Drifts, Stability and equilibrium problems.

Recommended Books:

- Wolfram, S. (2019). Cellular automaton fluids 1: Basic theory. In *Lattice Gas Methods for Partial Differential Equations* (pp. 19-74). CRC Press.
- Birk, L. (2019). *Fundamentals of ship hydrodynamics: Fluid mechanics, ship resistance and propulsion*. John Wiley & Sons.
- Muller, H. (2019). *Fluid sealing technology: principles and applications*. Routledge.
- Han, J. C., & Wright, L. (2020). *Experimental Methods in Heat Transfer and Fluid Mechanics*. CRC Press.

Course Name: Banach Algebra	**Course Code: MTH-719**
Course Structure: Lectures: 3	**Credit Hours:** 3

Course Outline:

Banach, Algebras, Ideals, Homomorphisms, Quotient Algebras, Wiener's Lemma, Gelfand's Theory of commutative Banach Algebras, The Notion of Gelfand's Topology, Radicals, Gelfand's Transforms. Basic properties of Spectra, Gelfand-Mazur Theorm, Symbolic Calculus, Differntiation, analytic Functions, Integration of A-valued Functions, Normed rings, Gelfand-Naimark Theorem.

Recommended Books:

Escassut, A. (2022). *Banach Algebras of Ultrametric Functions*. World Scientific.

Cianchi, A., Pick, L., & Slavíková, L. (2019). Banach algebras of weakly differentiable functions. *Journal d'Analyse Mathématique, 138*(2), 473-511.

Course Name: Computational Fluid Dynamics	**Course Code: MTH-720**
Course Structure: Lectures: 3	**Credit Hours:** 3

Course Outline:

Philosophy of Computational Fluid Dynamics, Computational Fluid Dynamics: Why?, Incompressible plane flows, Stream function and vorticity equations, Conservative form and normalizing systems, Method for solving vorticity transport equation, Basic finite difference forms, Conservative property, Convergence and stability analysis, Explicit and implicit methods, Stream function equation and boundary conditions, Schemes for advective diffusion equation, Upwind differencing and artificial vorticity, Solution for primitive variables.

Recommended Books:

- Tu, J., Yeoh, G. H., Liu, C., & Tao, Y. (2023). *Computational fluid dynamics: a practical approach*. Elsevier.
- Zikanov, O. (2019). *Essential computational fluid dynamics*. John Wiley & Sons.
- Rodriguez, S. (2019). *Applied Computational Fluid Dynamics and Turbulence Modeling: Practical Tools, Tips and Techniques*. Springer Nature.
- Sharma, A. (2021). *Introduction to computational fluid dynamics: development, application and analysis*. Springer Nature.
- Ferziger, J. H., Perić, M., & Street, R. L. (2019). *Computational methods for fluid dynamics*. springer.

Course Name: Cosmology	**Course Code: MTH-721**
Course Structure: Lectures: 3	**Credit Hours:** 3

Course Outline:

Review of Relativity. Historical background: Astronomy; Astrophysics; Cosmology. The cosmological principle and its strong form. The Einstein and DeSitter universe models. Measurement of cosmic distances. The Hubble law and the Friedmann models. Steady state models. The hot big bang model. The microwave background. Discussion of significance of a start of time. Fundamentals of high energy physics. The chronology and composition of the Universe. Non-baryonic dark matter. Problems of the standard model of cosmology. Bianchi spacetimes. Mixmaster models. Inflationary cosmology. Further developments of inflationary models. Kaluza-Klein cosmologies. Review of material.

Recommended Books:

- Baumann, D. (2022). *Cosmology*. Cambridge University Press.
- Năstase, H. (2019). *Cosmology and string theory* (Vol. 197, pp. 95-109). Cham: Springer.
- Elizalde, E. (2021). *The True Story of Modern Cosmology: origins, main actors and breakthroughs*. Springer Nature.
- Davis, A. M., Teixeira, M. T., & Schwartz, W. A. (Eds.). (2021). *Process Cosmology: New Integrations in Science and Philosophy*. Springer Nature.

Course Name: Claasical Theory of Fields	**Course Code: MTH-722**
Course Structure: Lectures: 3	**Credit Hours:** 3

Course Outline:

Review of continuum mechanics; solid and fluid media; constitutive equations and conservation equations. The concept of a field. The four dimensional formulation of fields and the stress-energy momentum tensor. The scalar field. Linear scalar fields and the Klein-Gordon equation. Non-linear scalar fields and fluids. The vector field. Linear massless scalar fields and the Maxwell field equations. The electromagnetic energy-momentum tensor. Electromagnetic waves. Diffraction of waves. Advanced and retarded potentials. Multipole expansion of the radiation field. The massive vector (Proca) field. The tensor field. The massless tensor field and Einstein field equations. Gravitational waves. The massive tensor field. Coupled field equations.

Recommended Books:

- Shifman, M. (2022). *Advanced topics in quantum field theory: A lecture course*. Cambridge University Press.
- Rohrlich, F. (2020). *Classical charged particles*. CRC Press.
- Dajczer, M., & Tojeiro, R. (2019). *Submanifold theory*. Springer US.
- Schwinger, J., DeRaad Jr, L. L., Milton, K., & Tsai, W. Y. (2019). *Classical electrodynamics*. CRC Press.

Course Name: **Computer Vision(variational)**	**Course Code: MTH-723**
Course Structure: Lectures: 3	**Credit Hours:** 3

Course Outline:

Introduction to the theory and applications of computer vision. Topics include: image representation,image segmentation, image analysis by mathematical morphology, texture, shape analysis and 3D version

Recommended Books:

- Szeliski, R. (2022). *Computer vision: algorithms and applications*. Springer Nature.
- Jain, S., & Paul, S. (Eds.). (2020). *Recent trends in image and signal processing in computer vision*. Singapore: Springer.
- Bouguila, N. (2022). *Hidden Markov Models and Applications*. W. Fan, & M. Amayri (Eds.). Springer.

Course Name: **Differential Geometry**	**Course Code: MTH-724**
Course Structure: Lectures: 3	**Credit Hours:** 3

Course Outline:

Manifolds: Definition, examples, importance and applications, Tensor and its basic algebra, Dimension of manifold, Tangent and cotangent vectors, Sub manifolds, Topology of manifolds. Tensors: Definition, examples, importance and applications, Tensor and its basic algebra, Differential forms, Tensors and a point. Tensor components, Order and rank of tensors, Tensor field. Some fundamental operations with tensors, Cotravariant and covariant tensors. Lie Groups, Geodesics, Curvature, Integration on Manifolds: Orientation of Manifolds, Integrals of forms.

Recommended Books:

1 Banchoff, T. F., & Lovett, S. (2022). *Differential geometry of curves and surfaces*. CRC Press.

2 Umehara, M., & Yamada, K. (2017). *Differential geometry of curves and surfaces*.

3

4 Do Carmo, M. P. (2016). *Differential geometry of curves and surfaces: revised and updated second edition*. Courier Dover Publications.

5 Kolár, I., Michor, P. W., & Slovák, J. (2013). *Natural operations in differential geometry*. Springer Science & Business Media.

6. Tapp, K. (2016). *Differential geometry of curves and surfaces*. Berlin: Springer.Dieudonné, J. (2020). History of Functional Analyais. In *Functional Analysis, Holomorphy, and Approximation Theory* (pp. 119-129). CRC Press.

Course Name: Digital Image Processing	**Course Code: MTH-725**
Course Structure: Lectures: 3	**Credit Hours:** 3

Course Outline:

Introduction to the theory and applications of 2-D signal and image processing: 2-D signals and system analysis, 2-D sampling and quantization, 2-D signals and image transforms, 2-D FIR filter design: image formation; image enhancement; image restoration; image coding; image reconstruction from projections; image compression; color image processing; current applications.

Recommended Books:

- Dumka, A., Ashok, A., Verma, P., Verma, P. (2020). Advanced Digital Image Processing and Its Applications in Big Data. United Kingdom: CRC Press.
- Baskar, A., Paaskar, A., Rajappa, M., Vasudevan, S. K., Murugesh, T. S. (2023). Digital Image Processing. United States: A Chapman & Hall Book/CRC Press, Taylor & Francis Group.
- Tyagi, V. (2021). Understanding Digital Image Processing. United Kingdom: Taylor & Francis Group.

Course Name: Elastodynamics	**Course Code: MTH-726**
Course Structure: Lectures: 3	**Credit Hours:** 3

Course Outline:

Waves in infinite media. Half-space problems; Surface waves. Dispersive media. Diffraction and scattering due to irregular structures.

Recommended Books:

- Grigorenko, A. Y., Müller, W. H., Loza, I. A. (2021). Selected Problems in the Elastodynamics of Piezoceramic Bodies. Germany: Springer International Publishing.
- Thorne, K. S., Blandford, R. D. (2021). Elasticity and Fluid Dynamics: Volume 3 of

Modern Classical Physics. United Kingdom: Princeton University Press.

- Fröhlich, J. (2022). A Segregated Finite Element Method for Cardiac Elastodynamics in a Fully Coupled Human Heart Model. Germany: Karlsruher Institut für Technologie (KIT).

Course Name: Electrodynamics-I	**Course Code: MTH-727**
Course Structure: Lectures: 3	**Credit Hours:** 3

Course Outline:

Maxwell's equations, Electromagnetic wave equation, Boundary conditions, Waves in conducting and non-conducting media, Reflection and polarization, Energy density and energy flux, Lorentz formula, Wave guides and cavity Resonators, Spherical and cylinderical waves, Inhomogeneous wave equation, Retarded potentials, Lenard-Wiechart potentials, Field of uniformly moving point charge, Radiation from a gruop of moving

Recommended Books:

- Jancewicz, B. (2022). Directed Quantities in Electrodynamics. Switzerland: Springer International Publishing.
- Lechner, K. (2018). Classical Electrodynamics: A Modern Perspective. Germany: Springer International Publishing.

Course Name: Electrodynamics-II	**Course Code: MTH-728**
Course Structure: Lectures: 3	**Credit Hours:** 3

Course Outline:

General angular and frequency distributions of radiation from accelerated charges, Thomson scattering, Cherenkov radiation, Fields and radiation of localized oscillating sources, Electric dipole fields and radiation, Magnetic dipole and electric quadruple fields, Multipole fields, Multipole expansion of the electromagnetic fields; Angular distributions sources of multipole radiation; Spherical wave expansion of a vector plane wave; Scattering of electromagnetic wave by a conducting sphere.

Recommended Books:

- Jancewicz, B. (2022). Directed Quantities in Electrodynamics. Switzerland: Springer International Publishing.
- Lechner, K. (2018). Classical Electrodynamics: A Modern

Perspective. Germany: Springer International Publishing., *and Approximation Theory* (pp. 119-129). CRC Press.

Course Name: **Fixed Point Theory**	**Course Code: MTH-729**
Course Structure: Lectures: 3	**Credit Hours:** 3

Course Outline:

Banach's contraction principle, Nonexpansive mappings, Sequential approximation techniques for nonexpansive mappings, Properties of fixed-point sets and minimal set, Multivalued mappings, Brouwer's fixed point theorem.

Recommended Books:

- Wu, H. (2020). Fixed Point Theory and Related Topics. (n.p.): MDPI AG.Zeidler, E. (2012).
- Metric Fixed Point Theory: Applications in Science, Engineering and Behavioural Sciences. (2022). Singapore: Springer Nature Singapore.

Course Name: **Field Theory**	**Course Code: MTH-730**
Course Structure: Lectures: 3	**Credit Hours:** 3

Course Outline:

Introduction, Field Extensions, Ruler and compass constructions, foundations of Galois Theory, Normality and stability, Splitting fields, Radical Extensions, The Trace and norm Theorems, Finite Fields, Simple Extensions, Cubic and Quadratic Equations.

Recommended Books:

- Burgess, C. P. (2020). Introduction to Effective Field Theory: Thinking Effectively about Hierarchies of Scale. United Kingdom: Cambridge University Press.
- Fine, B., Gaglione, A. M., Rosenberger, G. (2014). Introduction to Abstract Algebra: From Rings, Numbers, Groups, and Fields to Polynomials and Galois Theory. United States: Johns Hopkins University Press.

Course Name: Gas Dynamics	**Course Code: MTH-731**
Course Structure: Lectures: 3	**Credit Hours:** 3

Course Outline:

Compressible flows. Mathematical and Thermodynamic methods of gas dynamics. Supersonic, subsonic and hypersonic flows. Shock waves. Method of characteristics. One dimensional gas flow. The intersection of surfaces of discontinuity. Interactions. Two-dimensional gas flow. Fluid dynamics of combustion. Detonation and Deflagration waves.

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Recommended Books:

- Rathakrishnan, E. (2019). Applied gas dynamics.
- Zucker, R. D., & Biblarz, O. (2019). *Fundamentals of gas dynamics*. John Wiley & Sons.
- Li, Z., Shen, J., Gerhard, O., & Clarke, J. P. (2022). Gas Dynamics in the Galaxy: Total Mass Distribution and the Bar Pattern Speed. *The Astrophysical Journal*, *925*(1), 71.
- Li, J., Zhang, T., & Yang, S. (2022). *The two-dimensional Riemann problem in gas dynamics*. Routledge.
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Course Name: Multigrid Method for Linear and Non-Linear PDEs	**Course Code: MTH-732**
Course Structure: Lectures: 3	**Credit Hours:** 3

Course Outline:

Stencil Notations for differential operators, Ingredients of Multigrid, Error smoothing procedure, Two-grid cycle, Multigrid components, Linear Multigrid cycle, Full Multigrid (FMG), Local Fourier Analysis (LFA), Solution of Poisson equation in 2D and 3D, Non-linear Multigrid.

Recommended Books:

- Bramble, J. H. (2019). *Multigrid methods*. Chapman and Hall/CRC.
- Henson, V. (2003). Multigrid methods nonlinear problems: an overview. *Computational imaging*.
- El Houda, N. N., Mohammed, B., Essaid, B., Ahmad, I., Ahmad, H., & Askar, S. (2023). Multigrid Methods for The Solution of Nonlinear Variational Inequalities. *European Journal of Pure and Applied Mathematics*, *16*(3), 1956-1969.
- Mulder, W. A. (2020). Numerical Methods, Multigrid. In *Encyclopedia of Solid Earth Geophysics: Living Edition* (pp. 1-6). Springer.

Course Name: Mathematical Modeling and Numerical Simulations	**Course Code: MTH-733**
Course Structure: Lectures: 3	**Credit Hours:** 3

Course Outline:

Model and its different types, Finite models, Statistical models, Stochastic models, Formulation of a model, Laws and conservation principles, Discrete and continuous models, Manipulation into its most respective form, Evaluation of a model. Case studies, Continuum model, Transport phenomena, Diffusion and air pollution models, Microwave heating, Communication and Information technology.

Recommended Books:

- Brauer, F., Castillo-Chavez, C., & Feng, Z. (2019). *Mathematical models in epidemiology* (Vol. 32). New York: Springer.
- Mooney, D. D., & Swift, R. J. (2021). *A course in mathematical modeling* (Vol. 13). American Mathematical Society.
- Ellahi, R., Sait, S. M., Shehzad, N., & Mobin, N. (2019). Numerical simulation and mathematical modeling of electro-osmotic Couette–Poiseuille flow of MHD power-law nanofluid with entropy generation. *Symmetry*, *11*(8), 1038.
- Baronas, R., Ivanauskas, F., & Kulys, J. (2021). *Mathematical modeling of biosensors*. Springer International Publishing.

Course Name: Mathematical Techniques for Boundary Value Problems	**Course Code: MTH-734**
Course Structure: Lectures: 3	**Credit Hours:** 3

Course Outline:

Green's function method with applications to wave-propagation. Perturbation method: regular and singular perturbation techniques with applications. Variational methods. A survey of transform techniques; Wiener-Hopf technique with applications to diffraction problems.

Recommended Books:

- Wechselberger, M. (2020). *Geometric singular perturbation theory beyond the standard form* (Vol. 6). New York: Springer.
- Behrndt, J., Hassi, S., & De Snoo, H. (2020). *Boundary value problems, Weyl functions, and differential operators* (p. 772). Springer Nature.
- Kythe, P. K. (2020). *An introduction to boundary element methods*. CRC press.
- Wyld, H. W., & Powell, G. (2020). *Mathematical methods for physics*. CRC Press.

Course Name: Magnetohydrodynamics-I	**Course Code: MTH-735**
Course Structure: Lectures: 3	**Credit Hours:** 3

Course Outline:

Basic Equations: Equations of electrodynamics, Equations of Fluid Dynamics, Ohm’s law equations of magnetohydrodynamics.
Motion of an Incompressible Fluid: Motion of a viscous electrically conducting fluid with linear current flow, steady state motion along a magnetic field, wave motion of an ideal fluid.
Small Amplitude MHD Waves: Magneto-sonic waves. Alfve’s waves, damping and excitation of MHD waves, characteristics lines and surfaces.
Simples Waves and Shock Waves in Magnetohydrodynamics: Kinds of simple waves, distortion of the profile of a simple wave, discontinuities, simple and shock waves in relativistic magnetohydrodynamics, stability and structure of shock waves, discontinuities in various quantities, piston problem, oblique shock waves.

Recommended Books:

- Antoniou, S. M. (2023). Magnetohydrodynamics. I. Reduction of Equations through Lie Symmetries.
- Webb, G. (2018). *Magnetohydrodynamics and fluid dynamics: Action principles and conservation laws* (Vol. 946). Springer.

- Beresnyak, A. (2019). MHD turbulence. *Living Reviews in Computational Astrophysics*, *5*(1), 2.
- Goedbloed, H., Goedbloed, J. P., Keppens, R., & Poedts, S. (2019). *Magnetohydrodynamics: Of Laboratory and Astrophysical Plasmas*. Cambridge University Press.

Course Name: Magnetohydrodynamics-II	**Course Code: MTH-736**
Course Structure: Lectures: 3	**Credit Hours:** 3

Course Outline:

Flow of Conducting Fluid Past Magnetized Bodies: Flow of an ideal fluid past magnetized bodies, Fluid of finite electrical conductivity flow past a magnetized body.
Dynamo Theories: Elsasser's Theory, Bullard's Theory, Earth's field Turbulent motion and dissipation, vorticity anology.
Ionized Gases: Effects of molecular structure, Currents in a fully ionized gas, partially ionized gases, interstellar fields, dissipation in hot and cool clouds.

Recommended Books:

- Beresnyak, A., & Lazarian, A. (2019). *Turbulence in magnetohydrodynamics* (Vol. 12). Walter de Gruyter GmbH & Co KG.
- Andersson, N., Hawke, I., Celora, T., & Comer, G. L. (2022). The physics of non-ideal general relativistic magnetohydrodynamics. *Monthly Notices of the Royal Astronomical Society*, *509*(3), 3737-3750.

Course Name: Mathematical Biology	**Course Code: MTH-737**
Course Structure: Lectures: 3	**Credit Hours:** 3

Course Outline:

Population dynamics. Growth and spatial spread of organisms. Fisher's equation. Epidemiology the spread of plagues. Reaction-Diffusion models: Turing mechanism for pattern formation. How the leopard got his spots (and sometimes stripes). Enzyme Kinetics and chemical reactions: Michaelis-Menten theory Hormone cycles, neuron-firing. Mass transport; Taylor dispersion.

Recommended Books:

- Argyros, I. K., & George, S. (2020). *Mathematical Modeling for the Solutions of Equations and Systems of Equations with Applications Vol. 4*. Nova Science Publication, New York.
- Rihan, F. A. (2021). *Delay differential equations and applications to biology*. Singapore: Springer.
- Broom, M., & Rychtář, J. (2022). *Game-theoretical models in biology*. Chapman and Hall/CRC.
- Gause, G. F. (2019). *The struggle for existence: a classic of mathematical biology and ecology*. Courier Dover Publications.

Course Name: Numerical Solutions of Ordinary Differential Equations	**Course Code: MTH-738**
Course Structure: Lectures: 3	**Credit Hours:** 3

Course Outline:

Boundary and initial conditions, Polynomial approximations in higher dimensions.

Finite Element Method: The Galerkin method in one and more dimensions , Error bound on the Galarki method, The method of collocation, Error bounds on the collocation method, Comparison of efficiency of the finite difference and finite element method.

Finite Difference Method: Finite difference approximations.

Application to solution of linear and non-linear Partial Differential Equations appearing in Physical Problems.Euler's MethodConvergence of Euler's Method, Runge-Kutta method, Higher order Runge-Kutta formulas and their applications, Method of Taylor Expensions and their applications

Recommended Books:

- Epperson, J. F. (2021). *An introduction to numerical methods and analysis*. John Wiley & Sons.
- Yang, W. Y., Cao, W., Kim, J., Park, K. W., Park, H. H., Joung, J., ... & Im, T. (2020). *Applied numerical methods using MATLAB*. John Wiley & Sons.
- van Kan, J. J. I. M., Segal, A., & Vermolen, F. J. (2023). Numerical methods in scientific computing.
- Zwillinger, D., & Dobrushkin, V. (2021). *Handbook of differential equations*. CRC

Press.
- Salsa, S., & Verzini, G. (2022). *Partial differential equations in action: from modelling to theory* (Vol. 147). Springer Nature.

Course Name: Numerical Solutions of PDE's	**Course Code: MTH-739**
Course Structure: Lectures: 3	**Credit Hours:** 3

Course Outline:

Parabolic Equation: Explicit Finite Difference approximation, Crank Nickleson Implicit method: Derivative Boundary Conditions, The local truncation Errors, Stability. The Necessary and sufficient condition for stability, Finite difference approximation in spherical polar coordinate Hyperbolic equations, Analytical solution of Quasi Linear equations, Finite Difference method on a rectangular mesh for first order equations, Reduction of a first order equation to a system of ordinary differential equation: second order quasi-linear hyperbolic equation; Finite difference method on a rectangular mesh for second order equation ; simultaneous first order equations and Elliptic equation. Finite difference in polar coordinates; formulae for derivatives near curved boundaries; Improvement of the accuracy of the solution ; System iterative method for large linear systems, Jacobian , Gauss and SOR methods, Necessary and Sufficient condition for convergence of iterative methods , methods for acceleration convergence, the Gauss Seidel iteration matrix.

Recommended Books:

- Kuehn, C. (2019). *PDE dynamics: an introduction* (Vol. 23). SIAM.
- Constanda, C. (2022). *Solution techniques for elementary partial differential equations*. CRC press.
- Kuehn, C. (2019). *PDE dynamics: an introduction* (Vol. 23). SIAM.
- Chakraverty, S., Mahato, N., Karunakar, P., & Rao, T. D. (2019). *Advanced numerical and semi-analytical methods for differential equations*. John Wiley & Sons.
- Garabedian, P. R. (2023). *Partial differential equations* (Vol. 325). American Mathematical Society.
- *equations in action: from modelling to theory* (Vol. 147). Springer Nature.

Course Name: Numerical Solutions for Integral Equations	**Course Code: MTH-740**
Course Structure: Lectures: 3	**Credit Hours:** 3

Course Outline:

Linear integral equations of first and second kinds, Solution of integral equations of seconds kind by successive substitutions. The Fredholm theory and its applications Hillbert, Schmidt Theory of Integral Equations.

Recommended Books:

- Chew, W., Tong, M. S., & Bin, H. U. (2022). *Integral equation methods for electromagnetic and elastic waves*. Springer Nature.
- Cotta, R. M. (2020). *Integral transforms in computational heat and fluid flow*. CRC Press.
- Epperson, J. F. (2021). *An introduction to numerical methods and analysis*. John Wiley & Sons.
- Mandal, B. N., & Mandal, N. (2022). *Advances in dual integral equations*. Routledge.
- Kythe, P. K. (2020). *An introduction to boundary element methods*. CRC press.

Course Name: Numerical Linear Algebra	**Course Code: MTH-741**
Course Structure: Lectures: 3	**Credit Hours:** 3

Course Outline:

Matrix-Vector operations, Orthogonal vectors and matrices, Matrix and vector norms, Singular value, decomposition (SVD), Projectors and QR factorization Gram- Schmidt orthogonalization process, Householder triangularization, Least- squares, problems, Condition numbers, Gaussian elimination, and LU factorization, Pivoting and LUP factorization, Stability of Gaussian elimination, Cholesky Factorization, Overview of eigenvalue problems, Reduction to upper-Hessenberg Tridiagonal form, P ower and inverse power iteration, QR algorithm without shifts, QR algorithm with shifts, Arnoldi iteration, GMRES method, Lanczos iteration. Orthogonal polynomials and Gauss quadrature, Conjugate gradient (CG) method, Bi-Orthogonalization method.

Recommended Books:

- Lloyd Trefethen and David Bau (2019), Numerical Linear Algebra, SIAM.
- Bornemann, F. (2018). Numerical Linear Algebra: A Concise Introduction with MATLAB and Julia. Germany: Springer International Publishing.
- Trefethen, L. N. (2022). Numerical Linear Algebra: Twenty-Fifth Anniversary

Edition. United States: SIAM, Society for Industrial and Applied Mathematics.

Course Name: Non-Newtomian Fluid Mechanics	**Course Code: MTH-742**
Course Structure: Lectures: 3	**Credit Hours:** 3

Course Outline:

Classification of Non-Newtonian Fluids, Rheological formulae (Time-independent fluids, Thixotropic fluids, and viscoelastic fluids), Variable viscosity fluids, Cross viscosity fluids, The deformation rate, Viscoelastic equation, Materials with short memories, Time dependent viscosity. The Rivlin-Ericksen fluid, Basic equations of motion in rheological models. The linear viscoelastic liquid, Couette flow, Poiseuille flows. The current semi-infinite field, Axial oscillatory tube flow, Angular oscillatory motion, Periodic transients, Basic equations in boundary layer theory, Orders of magnitude, Truncated solutions for viscoelastic flow, Similarity solutions, Turbulent boundary layers, Stability analysis.

Recommended Books:

- Farina, A., Mikelić, A., Fusi, L., Saccomandi, G., Sequeira, A., Toro, E. F. (2018). Non-Newtonian Fluid Mechanics and Complex Flows: Levico Terme, It: Springer International Publishing.
- Spurk, J. H., Aksel, N. (2019). Fluid Mechanics. Germany: Springer International Publishing.

Course Name: Parallel Algorithms	**Course Code: MTH-743**
Course Structure: Lectures: 3	**Credit Hours:** 3

Course Outline:

Theoretical models of parallel computation: Variants of the PRAM model. Performance of parallel algorithms. Basic Techniques: Balanced trees, recursive doubling, divide and conquer, partitioning. Pipe lining, accelerated cascading, symmetry breaking. List ranking, the euler tour techniques, tree contraction. Algorithms for searching, merging and sorting. Graph algorithms: Connected components, colouring. Parallel algorithms on interconnection networks and other architectures. Limits to parallelisability. P-completeness.

Recommended Books:

- Trobec, R., Slivnik, B., Bulić, P., Robič, B. (2018). *Introduction to Parallel Computing*: From Algorithms to Programming on State-of-the-Art Platforms. Germany: Springer International Publishing.

- Sanders, P., Mehlhorn, K., Dietzfelbinger, M., Dementiev, R. (2019). Sequential and Parallel Algorithms and Data Structures: The Basic Toolbox. Germany: Springer International Publishing.

Course Name: Plasma Theory-I	**Course Code: MTH-744**
Course Structure: Lectures: 3	**Credit Hours:** 3

Course Outline:

Introduction: Definition of plasma; temperature; Debye shielding, the plasma parameter; criteria for plasmas; introduction to controlled fusion.

Fluid description of plasma: Wave propagation in plasma; derivation of dispersion relations for simple electrostatic and electromagnetic modes.

Equilibrium and stability (with fluid model); Hydromagnetic equilibrium/diffusion of the magnetic field into a plasma; classification of instabilities; two-stream instability; gravitational instability; resistive drift waves.

Space plasma: Atmospheric source of magnetospheric plasma and its temperature; plasma from Jupiter.

Recommended Books:

- Goldston, R., Rutherford, P. (2020). Introduction to Plasma Physics. United Kingdom: Taylor & Francis.
- Gurnett, D. A., Bhattacharjee, A. (2017). Introduction to Plasma Physics: With Space, Laboratory and Astrophysical Applications. United Kingdom: Cambridge University Press.
- Ichimaru, S. (2018). Basic Principles of Plasma Physics: A Statistical Approach. United States: CRC Press.
- Conde, L., Conde, C. L. (2020). An Introduction to Plasma Physics and Its Space Applications: Volume 2. United Kingdom: IOP Publishing.

Course Name: Plasma Theory-II	**Course Code: MTH-745**
Course Structure: Lectures: 3	**Credit Hours:** 3

Course Outline:

The Plasma Theory of Waves: Solution of localized Vlasov equation Vlasov theory of small amplitude waves in field free uniform/nonuniform magnetized cold/hot plasmas; the theory of instability.

The nonlinear Vlasov theory of plasma waves and instabilities: Conservation of particles, momentum and energy in quasilinear theory; Landau damping; the gentle-bump and two-stream instability in quasilinear theory; plasma wave echoes; nonlinear wave-particle interaction. **Fluctuations, correlations and radiations:** Shielding of a moving test charge, electric field fluctuations in maxwellian and nonmaxwellian plasmas, emission of electrostatic waves; electromagnetic fluctuations, emission of radiation from plasma; black body radiation; cyclotron radiation.

Recommended Books:

- Ichimaru, S. (2018). Basic Principles of Plasma Physics: A Statistical Approach. United States: CRC Press.
- Conde, L., Conde, C. L. (2020). An Introduction to Plasma Physics and Its Space Applications: Volume 2. United Kingdom: IOP Publishing.

Course Name: Perturbation Methods-I	**Course Code: MTH-746**
Course Structure: Lectures: 3	**Credit Hours:** 3

Course Outline:

Difference equations, Dimensional analysis, Expansions, Approximate solutions of linear differential equations, order symbols, Asymptotic series, Quadratic and cubic algebraic equations and its solutions by perturbation method, Straightforward expansion, Lindsted-Poincar Technique, Method of Renormalization, Method of multiple scales, dominant balance method, WKB method, Method of strained parameters.

Recommended Books:

- Jazar, R. N. (2021). Perturbation Methods in Science and Engineering. Switzerland: Springer International Publishing.
- Cao, D., Peng, S., Yan, S. (2021). Singularly Perturbed

Course Name: Perturbation Methods-II	**Course Code: MTH-747**
Course Structure: Lectures: 3	**Credit Hours:** 3

Course Outline:

Regular perturbation, Singular perturbation, Boundary layer, The method of matched asymptotic expansion, equations with large parameter, Solution of partial differential equations by perturbation methods, Asymptotic expansion of integrals Laplace's method, Watson's Lemma, Riemann-Lebesgue lemma.

Recommended Books:

- Bazaar, R. N. (2021). Perturbation Methods in Science and Engineering. Switzerland: Springer International Publishing.
- Turfus, C. (2021). Perturbation Methods in Credit Derivatives: Strategies for Efficient Risk Management. United Kingdom: Wiley.

Course Name: Rings and Categories of Modules	**Course Code: MTH-748**
Course Structure: Lectures: 3	**Credit Hours:** 3

Course Outline:

Rings and Subrings, ring Homomorphism, Ideals and Factor rings, some special rings, polynomial rings, product and functions ring, centre of a ring, the opposite ring of a ring, Matrix ring, endomorphism rings, Idempotents, Nilpotent elements.

Modules and Sub modules, linear combinations and sub modules, Factor modules, change of rings, Annihilators, homomorphism of modules, Epimorphisms and Monomorphisms, Factor theorem, Isomorphism theorem, Exactness, Category of modules, Endomorphism rings, direct Summands, Split Homomorphism, Projections, Idempotents Endomorphisms, Essential and superfluous sub modules, Direct sum and product of modules. Direct sums-coproduct, Internal direct sums, decomposition of rings.

Simple modules, Semi simple modules, socle, radical, finitely generated modules, role of radical and socle, artinian, noetherian ring.

Recommended Books:

- Wang, F., Kim, H. (2017). Foundations of Commutative Rings and Their Modules. Singapore: Springer Nature Singapore.
- Wisbauer, R. (2018*)*. Foundations of Module and Ring Theory. United Kingdom: CRC Press.

Course Name: Riemann Surfaces	**Course Code: MTH-749**
Course Structure: Lectures: 3	**Credit Hours:** 3

Course Outline:

Introduction to the theory of Riemann surfaces. The Riemann surface of an analytic function. Covering surfaces, monodromy theorem, groups of cover transformations, uniformization theorem. Differentials and integrals, Riemann-Roch theorem.

Recommended Books:

- Cavalieri, R., Miles, E. (2016). Riemann Surfaces and Algebraic Curves: A First Course in Hurwitz Theory. United Kingdom: Cambridge University Press.
- Natanzon, S. M. (2020). Complex Analysis, Riemann Surfaces and Integrable Systems. Germany: Springer International Publishing.
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Course Name: **Research Methodology**	**Course Code: MTH-750**
Course Structure: Lectures: 3	**Credit Hours:** 3

Course Outline:

This module is an introduction to the research methods and methodology generally used in mathematics. This module will enable students to review critically and evaluate the scientific writing from books and research papers (both pure and applied) in the following contents:

- Purpose of Research
- Different types of research (Experimental, non-experimental)
- Sources of data
- Research aims and objectives
- Research design
- Problem identification and hypothesis
- How to review literature and avoid plagiarism
- writing research proposal

Recommended Books:

- Kumar, R. (2018). Research Methodology: A Step By Step Guide For Beginners, 2/E. India: Pearson Education.
- Thomas, C. G. (2021). Research Methodology and Scientific Writing. Germany: Springer International Publishing

Course Name: Regression Analysis.	**Course Code: MTH-751**
Course Structure: Lectures: 3	**Credit Hours:** 3

Course Outline:

Simple linear regression analysis: The two variable linear model and assumptions. The ordinary le squares estimators and their properties. Inference in the linear model.

The general linear Model : Introduction . Assumptions and their role. The ordinary least squares estimators. Gauss-markov's theorem. Inference in the ordinary least squares model. Use of extran information in the linear regression model. Least squares method subject to linear restrictions. In about the set of linear hypotheses. Problems in the general linear model.

Multocollinearity and its consequence : Detection and its remedies. Ridge Regression. Heterosced and its sources. The general least squares methods and its properties. Testing of Heteroscedasticity. Autocorrelation and its consequences for ordinary least squares estimators. Durbin-waston test for Autocorrelation. Dummy variables and their role. Distributed large models difficulties involved in estimation. Specification error. Exclusion of relevant variables and inclusi of irrelevant variables.

Errors in variables.

Recommended Books:

- Darlington, R. B., Hayes, A. F. (2017). Regression Analysis and Linear Models: Concepts, Applications, and Implementation. United Kingdom: Guilford Publication.
- Olive, D. J. (2017). Linear Regression. Germany: Springer International Publishing.
- Gujarati, D. N. (2018). Linear Regression: A mathematical Introduction. United States:

SAGE Publications.
- Thrane, C. (2020). Applied Regression Analysis: Doing, Interpreting and Reporting. United Kingdom: Routledge.
-

Course Name: **Spectral Theory in Hilbert Spaces**	**Course Code: MTH-752**
Course Structure: Lectures: 3	**Credit Hours:** 3

Course Outline:

The Concepts of Hilbert Spaces, Inner Product Spaces, Hilbert Spaces, Bounded Liner operators, Bilinear Forms, Adjoint Operators, Projection Operators, The Fourier-Phincherl Operator. General Theory and Special Analysis of Linear Operators, closed Linear Operators, Invariant Subspaces of a Linear Operator, Eigenvalues and Spectrum of a Linear Operator, The Special Decomposition of a Bounded Self-Adjoint Operator.

Recommended Books:
- Borthwick, D. (2020). Spectral Theory: Basic Concepts and Applications. Germany: Springer International Publishing.
- Halmos, P. R. (2017). Introduction to Hilbert Space and the Theory of Spectral Multiplicity: Second Edition. United States: Dover Publications.
-

Course Name: **: Statistical Decision Theory**	**Course Code: MTH-753**
Course Structure: Lectures: 3	**Credit Hours:** 3

Course Outline:

Decision functions, Risk functions, Utility and subjective probability, Randomization, Optimal decision rules. Admissibility and completeness, Existence of Bayes decision rules, Existence of a minimal complete class, Essential completeness of the class of nonrandomized rules. The minimax theorem. Invariant statistical decision problems. Multiple decision problems. Sequential Decision problems.

Recommended Books:
- Zabarankin, M., Uryasev, S. (2016). Statistical Decision Problems: Selected Concepts and Portfolio Safeguard Case Studies. United States: Springer New York.
- Longford, N. T. (2021). Statistics for Making Decisions. United States: CRC Press.

Course Name: **Several Complex Variables**	**Course Code: MTH-754**
Course Structure: Lectures: 3	**Credit Hours:** 3

Course Outline:

Power series in several complex variables, domains of holomorphy, pseudo convexity, plurisubharmonic functions, the Levi problem. Domains with smooth boundary, tangential Cauchy-Riemann equations, the Lewy and Bochner extension theorems. The $\overlin e {\partial }$-operator and Hartog's Theorem, Dol beault-Grothendieck lemma, theorems of Runge, Mittag-Leffler and Weierstrass. Analytic continuation, monodromy theorem, uniformization and Koebe's theorem, discontinuous groups.

Recommended Books:

- Robert C. Gunning., Hugo Rossi. (2022). Analytic Functions of Several Complex Variables. United States: American Mathematical Society.
- Krantz, S. G. (2017). Harmonic and Complex Analysis in Several Variables. Germany: Springer International Publishing.

Course Name: **Topological Vector Spaces**	**Course Code: MTH-755**
Course Structure: Lectures: 3	**Credit Hours:** 3

Course Outline:

Vector Spaces, Convex sets, Balanced sets, Absorbent sets, Linear Functionals, Linear Manifolds, Sublinear Functionals and Extension of Linear Functionals. Topological Vector Spaces, Definitions and General properties, Product and Quotient Spaces, Finite Dimensional Topological vector spaces, Closed Hyperplanes, Bounded Sets, Convex sets and Compact sets in Topological Vector Spaces, Seminorms, Locally Convex Spaces, Minkowski Functionals, Normable Spaces, Metrizable Topological Linear Spaces.

Recommended Books:

- Voigt, J. (2020). A Course on Topological Vector Spaces. Germany: Springer International Publishing.
- Bogachev, V., Smolyanov, O. (2017). Topological Vector Spaces and Their Applications. Germany: Springer International Publishing.

Course Name: **Viscous Fluid-I**	**Course Code: MTH-756**
Course Structure: Lectures: 3	**Credit Hours:** 3

<u>Course Outline:</u>

Eulerian approach, Lagrangian description, Properties of fluids, Transport properties, Kinematic properties, thermodynamics properties, Boundary conditions for viscous flows and heat conducting flows problems, Conservation of mass (equation of continuity), conservation of momentum (equations of Navier-Stokes equations), conservation of energy (energy equations), Dimensionalization and dimensionless parameters in viscous flow, Vorticity transport equation, Stream function, Steady flow,
Unsteady flow, creeping flow and boundary layer flow, Couette flows, Poiseuille flow, CouettePoiseuille

flow between parallel plates, Stokes first problem, Stokes second problem, Unsteady flow between two infinite plates, Asymptotic suction flows: uniform suction on a plane, flow between parallel plates with top suction and bottom injection.

<u>Recommended Books</u>:

- White, F. M., & Majdalani, J. (2006). *Viscous fluid flow* (Vol. 3, pp. 433-434). New York: McGraw-Hill.
- Schlichting, H., & Gersten, K. (2016). *Boundary-layer theory*. springer.
- Batchelor, G. K. (2000). An Introduction to Fluid Dynamics. *An Introduction to Fluid Dynamics*, 635.

Course Name: **Viscous Fluid-II**	**Course Code: MTH-757**
Course Structure: Lectures: 3	**Credit Hours:** 3

<u>Course Outline:</u>

Similarity solution, Berman problem, Plane stagnation flow, axisymmetric stagnation flow, flow near an infinite rotating disk, Jeffery Hammel flow in a wedge shaped region and it solution for

small wedge angle, Stokes solution for an immersed sphere, Derivation of boundnry-Iayer equations for two-dimensional flow, The laminar boundary layer equations, The approximate method due to the von Karman and K. Pohlhausen for two dimensional flows, Blasius problem of flat plate flow, Falker-Skan wedge flows, Heat transfer for Falker-Skan flows, two dimensional steady free convection, viscous flows over a stretching sheet, thin film flows
Recommended Books: • White, F. M., & Majdalani, J. (2006). *Viscous fluid flow* (Vol. 3, pp. 433-434). New York: McGraw-Hill. • Schlichting, H., & Gersten, K. (2016). *Boundary-layer theory*. springer. • Batchelor, G. K. (2000). An Introduction to Fluid Dynamics. *An Introduction to Fluid Dynamics*, 635.

Course Name: Fractional Calculus	**Course Code: MTH-758**
Course Structure: Lectures: 3	**Credit Hours:** 3

Course Outline:

Special functions, The Gamma function, The Beta function, The Mittage-Leffler function, the Meline Rose function and the Wright function. Difference operator, forward and backward operators. Historical origins of fractional calculus. Introduction to fractional order derivatives and integrals. Riemann, Liouville-Caputo fractional derivative-Riesz-Feller fractional derivative-Grunwal-Letnikov, Hadamard fractional derivative. The Riemann-Liouville, the Caputo's, types derivatives and their properties and relationship. Fractional order derivatives and integrals of polynomials, rational and trigonometric functions. The left and right fractional order derivatives. Basic properties of fractional order derivatives and integrals (Linearity, composition, etc.), Leibnitz's rules of fractional differentiations. Geometrical interpretation of fractional order derivatives and integrals. Laplace of fractional order derivatives and integrals. Fourier transform of fractional order derivative. Introduction to Milin's Transform and its applications.

Recommended Books:

- Kilbas, A. A., Srivastava, H. M., & Trujillo, J. J. (2006). *Theory and applications of fractional differential equations* (Vol. 204). elsevier.
- Zhou, Y. (2023). *Basic theory of fractional differential equations*. World scientific.
- Goufo, E. F. D., & Toudjeu, I. T. (2019). Analysis of recent fractional evolution equations and applications. *Chaos, Solitons & Fractals, 126*, 337-350.
- Diethelm, K., & Ford, N. J. (2010). The analysis of fractional differential equations. *Lect. Notes Math, 2004*, 3-12.

Course Name: **Fractional Differential Equations**	**Course Code: MTH-759**
Course Structure: Lectures: 3	**Credit Hours:** 3

Course Outline:

Introduction to Fractional Derivatives and Integrals, Solutions of Linear Fractional-order Equations by Laplace Transforms. Introduction to some fixed point theorems including Banach,

Schauder and Schaefer, etc for obtaining the Existence and Uniqueness results, formation of Green's Function, Other Methods for the Solution of Fractional-order Equations, existence theory for one, two and more solutions of Fractional-order equations and their Systems, Controllers, Survey of Applications of the Fractional Calculus.

Recommended Books:

- De Oliveira, E. C., & Tenreiro Machado, J. A. (2014). A review of definitions for fractional derivatives and integral. *Mathematical Problems in Engineering*, *2014*.
- Jin, B. (2021). *Fractional differential equations*. Springer International Publishing.
- Kilbas, A. A., Srivastava, H. M., & Trujillo, J. J. (2006). *Theory and applications of fractional differential equations* (Vol. 204). elsevier.
- Zhou, Y. (2023). *Basic theory of fractional differential equations*. World scientific.

Course Name: **Fuzzy Fixed-point theory**	**Course Code: MTH-760**
Course Structure: Lectures: 3	**Credit Hours:** 3

Course Outline:

Fuzzy set, Fuzzy metric space, F-convergence, F-Cauchy's sequence, F-complete fuzzy metric space, F-continuity, Fuzzy contractive mapping, Fuzzy contractive sequence, Banach contraction theorem in fuzzy metric spaces, Kramosil and Michalek's fuzzy metric spaces, Menger space, Probabilistic contraction, Banach contraction theorem for M-complete fuzzy metric spaces, Convergence in the sense of Fréchet, Edelstein fuzzy contractive mappings, R-weak commutatively of type (P) in fuzzy metric spaces, Compatible maps and compatible maps of types (α) and (β), common fixed point theorems for compatible maps of type (β) on fuzzy metric spaces, cyclic weak ϕ−contractions in fuzzy metric spaces, non-Archimedean fuzzy metric space.

Recommended Books:

- Gregori, V., López-Crevillén, A., Morillas, S., & Sapena, A. (2009). On convergence in fuzzy metric spaces. *Topology and its Applications*, *156*(18), 3002-3006.
- Gregori, V., & Sapena, A. (2018, June). Forty Years of Fuzzy Metrics: In Honour of Manuel López-Pellicer. In *The Meeting in Topology and Functional Analysis* (pp. 137-151). Cham: Springer International Publishing.
- Vasuki, R., & Veeramani, P. (2003). Fixed point theorems and Cauchy sequences in fuzzy metric spaces. *Fuzzy sets and systems*, *135*(3), 415-417.
- Miheţ, D. (2004). A Banach contraction theorem in fuzzy metric spaces. *Fuzzy sets and systems*, *144*(3), 431-439.
- Miheţ, D. (2007). On fuzzy contractive mappings in fuzzy metric spaces. *Fuzzy Sets and Systems*, *158*(8), 915-921.
-

Course Name: **Fuzzy Sets and Their Applications**	**Course Code: MTH-761**
Course Structure: Lectures: 3	**Credit Hours:** 3

Course Outline:

Classical sets via Fuzzy Sets, Need for fuzzy sets, Definition and Mathematical representations, Level Sets, Fuzzy functions, Zadeh's, Extension Principle.

Operations on [0, 1], Fuzzy negation, triangular norms, t-norms, fuzzy implications, Aggregation Operations, Fuzzy Functional Equations.

Fuzzy Binary and n-ary relations, composition of fuzzy relations, Fuzzy Equivalence Relations, Fuzzy Compatibility Relations, Fuzzy Relational Equations.

Fuzzy Measures, Evidence Theory, Necessity and Belief Measures, Probability Measures via Possibility Measures.

Fuzzy Decision Making, Fuzzy Relational Inference, Compositional Rule of Inference, Efficiency of Inference, Hierarchical.

Fuzzy If-Then Rule Base, Inference Engine, Takagi-Sugeno Fuzzy Systems, Function Approximation.

Recommended Books:

- Klir, G. J., & Yuan, B. (2015). Fuzzy Sets and Fuzzy Logic: Theory and Applications.
- Panigrahi, D. P., & Mujumdar, P. P. (2000). Reservoir operation modelling with fuzzy logic. *Water Resources Management*, *14*, 89-109.
- Passino, K. M., Yurkovich, S., & Reinfrank, M. (1998). *Fuzzy control* (Vol. 42, pp. 15-21). Reading, MA: Addison-wesley.
- Massanet, S., Fernandez-Peralta, R., Baczyński, M., & Jayaram, B. (2023). On valuable and troubling practices in the research on classes of fuzzy implication functions. *Fuzzy Sets and Systems*, 108786.
-

Course Name: Advanced Group Theory	**Course Code: MTH-801**
Course Structure: Lectures: 3	**Credit Hours:** 3

Course Outline:

Survey of group theory, (elementary concepts) symmetric and alternating groups of finite degree, orbit of symmetric and alternating groups, transitive group. Group action on sets, Group action on groups, periodic torsion free and mixed abelian groups, free abelian groups. Finitely generated groups, Defination of a modular lattices and distributive lattices, the lattices of all sub groups of a group, the lattice of admissible, sub group, the lattice of normal subgroups.

Recommended Books:

- Hall, M. (2018). *The theory of groups*. Courier Dover Publications.
- Scott, W. R. (2012). *Group theory*. Courier Corporation.
  Davidoff, G. P., Sarnak, P., & Valette, A. (2003). *Elementary number theory, group theory, and Ramanujan graphs* (Vol. 55, No. 1, pp. 45-80). Cambridge: Cambridge university press.

Course Name: Special Functions	**Course Code: MTH-802**
Course Structure: Lectures: 3	**Credit Hours:** 3

Course Outline:

Generalized Hyper geomeatrio Function.

The function pFq, the exponential and binomial functions, a differential equation, other salutions of the differential equation. The pFq, with unt argument. Saalschutz theorem. Whipple's theorem. The Barnes integrals and the function pFq.

Bessel Function.

Definition of Jn(z),Bessel's differential equation differential recurrence relations. A pure recurrence relation. Bessel's integral Index half an odd integer. Modified Bessel Functions. Neumann polynomials.

Confluent Hyper geometric Function.

Basic properties of the $1^{F}1$. Kummer's first formula. Kummer's second formula. Definition of the function Wk,m(z).

Recommended Books:

- Mathai, A. M., & Haubold, H. J. (2008). *Special functions for applied scientists* (Vol. 4). New York: Springer.
- Whittaker, E. T., & Watson, G. N. (1920). *A course of modern analysis: an introduction to the general theory of infinite processes and of analytic functions; with an account of the principal transcendental functions*. University Press.

Course Name: Advanced Special Functions	**Course Code: MTH-803**
Course Structure: Lectures: 3	**Credit Hours:** 3

Course Outline:

Orthogonal Polynomials.

Zero of orthogonal polynomials. The three term recurrence relation Christoffel Darbax formula.

Hermite Polynomials.

Decinition of $H_n(z)$ recurrence relations,Rodrigues formula; other generating functions.

Leguerre Polynamials.

The polynomials $L_n(\alpha)(X)$ Recurrence relations. The Rodrigues formula. Special properties. The simple Laguerrep Polynomials.

Elliptic Functions.

Deubly Periodic functions. Elliptic functions, elementary properties. Order of an elliptic function. The weierstrass function P (z). A differential equation for P (z). Connection with elliptic integrals.

Recommended Books:

- Mathai, A. M., & Haubold, H. J. (2008). *Special functions for applied scientists* (Vol. 4). New York: Springer.
- Whittaker, E. T., & Watson, G. N. (1920). *A course of modern analysis: an introduction*

to the general theory of infinite processes and of analytic functions; with an account of the principal transcendental functions. University Press.

theory, and Ramanujan graphs (Vol. 55, No. 1, pp. 45-80). Cambridge: Cambridge university press.

Course Name: Advanced Integrals Equations	**Course Code: MTH-804**
Course Structure: Lectures: 3	**Credit Hours:** 3

Course Outline:

Integral equations. Type of integral equations. Valtern Solutions. Fredholm equations.

Symmetric equations. Theory of Hibert-Schmidt. Symmetric Kernels and othognal systems of functions. Singular or Non-linear integral equations. Dirichlet's problem and its application. The biharmonic equation. Application of Green's Salution. The generalized method of schwerz. Certain applications of integrals analogous to potential. Application of the theory of symmetric integral equations. Certain applications of the theory of ingular integral equations. Hibert's problem.

Recommended Books:

- Moiseiwitsch, B. L. (2011). *Integral equations*. Courier Corporation.
- *Integral Equations, by F.G. Tricomi Inter Science 1992.*
- Polyanin, P., & Manzhirov, A. V. (2008). *Handbook of integral equations*. Chapman and Hall/CRC.
- Kanwal, R. P. (2013). *Linear integral equations*. Springer Science & Business Media.
- Colton, D., & Kress, R. (2013). *Integral equation methods in scattering theory*. Society for Industrial and Applied Mathematics..
- Matoog, R. T. (2014). On the solution of the Fredholm integral equation with hyper singular kernel. *Journal of American Science*, *10*(2).
- Luchko, Y. (2019). Some schemata for applications of the integral transforms of mathematical physics. *Mathematics*, *7*(3), 254..
- Gray, R. M., & Goodman, J. W. (2012). *Fourier transforms: an introduction for engineers* (Vol. 322). Springer Science & Business Media.
- Delves, L. M., & Mohamed, J. L. (1985). *Computational methods for integral equations*. CUP Archive.

Course Name: Nonlinear Dynamics and Chaos	**Course Code: MTH-805**
Course Structure: Lectures: 3	**Credit Hours:** 3

Course Outline:

Introduction in one and two Variables, Historical Survey of the development of Optimization Techniques, Algorithms that Involve Matrix Inversion, Gradient Techniques, Direct Search Techniques, The Constrained Problem, Mathematical Programming, The Allocation Problem, Geometric Programming.
The Calculus of Variations, Pontryagin's Maximum Principle, The solution of Optimal Path Problem By Dynamic Programming, Parametric Conversion, The maximum Principle applied by Iterating in Function Space.

Recommended Books:

- Strogatz, S. H. (1996). Nonlinear dynamics and chaos.
- Aguirre, L. A., & Letellier, C. (2009). Modeling nonlinear dynamics and chaos: A review. *Mathematical Problems in Engineering*, *2009*.

Course Name : Advanced General Relativity	**Course Code: MTH-806**
Course Structure: Lectures: 3	**Credit Hours:** 3

Course Outline:

Einstein's Field Equations and General Relativity, Einstein's Field Equation, The geometry and physics of Universe and their relation, Space-time and 4-dimensional manifold, Space-time and Tetrad. Homogenous and Isotropic Cosmology, Some Special solutions of EFE. The Schwarzschild solution, Schwarzschild Vacuum static space time, Derivation of the Schwarzschild solution, Interior solutions, Birkhoffs theorem, Stationary, Axisymmetric Solutions, Spatially homogenous Cosmologies, Algebraically Special Solutions Perturbations, Singularity, Singularity theorems. Time like and null geodesics, Congruences, Conjugate points, Existence of Maximum Length Curves. Black holes and the cosmic sensor conjecture, General properties of Black Holes, the Charged Kerr Black Hole.

Recommended Books:

- Wald, R. M. (2010). *General relativity*. University of Chicago press.
- Hawking, S. W., & Ellis, G. F. (2023). *The large scale structure of space-time*.

Cambridge university press.

Course Name: **Applied Data Analysis Techniques**	**Course Code: MTH-807**
Course Structure: Lectures: 3	**Credit Hours:** 3

Course Outline:

Introduction to model building Role of assumption, sharpness of inference, application parameter zing the model, parametric models simple, multiple regression model non linear regression model, ridge regression, robust regression. Logistic regression, probit, regression. Estimation (Model based) maximum likelihood estimation, Estimation (Methodology based) classical (parametric, semi perametric, baysien parametric) non-parametric. model, smoothing spline, kernel regression, estimation (Model based) kernel and smoothing methods.

Recommended Books:

- Gujarati, D. N. (2022). *Basic econometrics*. Prentice Hall.
- Greene, W. H. (2000). Econometric analysis 4th edition. *International edition, New Jersey: Prentice Hall*, 201-215.
- Johnson, R. A., & Wichern, D. W. (2002). Applied multivariate statistical analysis.
- Vittinghoff, E., Glidden, D. V., Shiboski, S. C., McCulloch, C. E., Vittinghoff, E., Glidden, D. V., ... & McCulloch, C. E. (2012). Logistic regression. *Regression methods in biostatistics: linear, logistic, survival, and repeated measures models*, 139-202.

Course Name: **Advanced Mathematical Physics**	**Course Code: MTH-808**
Course Structure: Lectures: 3	**Credit Hours:** 3

Course Outline:

Nonlinear ordinary differential equations, Bernoulli’s equation, Riccati equation, Lane-Emden equation, Nonlinear Pendulum, Duffing’s equation, Pinney’s equation, Perturbation theory, Bogoliubov-Krilov method. Linear partial differential equations, classification, initial and boundary values problems, Fourier analysis, Heat equation, Wave equation, Laplace equation etc. Integral equations, classification, integral transform separable kernels, singular integral equations, Wiencr-Hopf equations, Fredholm theory, series solutions. Variational methods, The Euler-Lagrange equations, Solutions to some famous problems, Sturm-Liouville Problem and variational principles, Rayleigh-Ritz Methods for partial differential equations. Matrix algebra, method of Faddeev, Caley-Hamilton' theorem function of matrices. Functions of matrices, Kronecker and Tensor product, special matrices

Recommended Books:

- Stephenson, G., & Radmore, P. M. (1990). *Advanced mathematical methods for engineering and science students*. Cambridge University Press.
- Riley, K. F., Hobson, M. P., & Bence, S. J. (2006). Mathematical Methods for Physics and Engineering: A Comprehensive Guide.

Course Name: Advanced Group Theory	**Course Code: MTH-801**
Course Structure: Lectures: 3	**Credit Hours:** 3

Course Outline:

Survey of group theory, (elementary concepts) symmetric and alternating groups of finite degree, orbit of symmetric and alternating groups, transitive group. Group action on sets, Group action on groups, periodic torsion free and mixed abelian groups, free abelian groups. Finitely generated groups, Defination of a modular lattices and distributive lattices, the lattices of all sub groups of a group, the lattice of admissible, sub group, the lattice of normal subgroups.

Recommended Books:

- Hall, M. (2018). *The theory of groups*. Courier Dover Publications.
- Scott, W. R. (2012). *Group theory*. Courier Corporation.

Davidoff, G. P., Sarnak, P., & Valette, A. (2003). *Elementary number theory, group theory, and Ramanujan graphs* (Vol. 55, No. 1, pp. 45-80). Cambridge: Cambridge university press.

Course Name: Advanced Mathematical Modeling	**Course Code: MTH-809**
Course Structure: Lectures: 3	**Credit Hours:** 3

Course Outline:

Modeling with Differential Equations: Exponential growth and decay. Linear, non-linear systems of differential equations. Modeling with integration. Modeling with random numbers: Simulating qualitative random variables. Simulating discrete random variables. Standard models. Monte Carlo simulation. Fitting models to data. Bilinear interpolation and Coons patch.

Recommended Books:

- Edwards, D., Hamson, M., Edwards, D., & Hamson, M. (1996). Setting up Models. *Mathematical Modelling Skills*, 18-28.
- Giordano, F. R., Weir, M. D., & Fox, W. P. (2003). Mathematical modeling. *Thomson-Brookes/Cole*.
- Law, A. M., Kelton, W. D., & Kelton, W. D. (2007). *Simulation modeling and analysis* (Vol. 3). New York: Mcgraw-hill.
- Paul, R. J. (1992, December). The computer aided simulation modeling environment: an overview. In *Proceedings of the 24th conference on Winter simulation* (pp. 737-746).
- Aris, R. (2012). *Mathematical Modelling Techniques*. Courier Corporation.

Course Name: Nonlinear Optimization	**Course Code: MTH-810**
Course Structure: Lectures: 3	**Credit Hours:** 3

Course Outline:

Implications of nonlinearity, dynamics and chaos, The role of dimensionality, One-dimensional systems, One dimensional flow: visualizing the solution space, Stability and fixed points, Linear stability analysis. Existence and uniqueness, Applications and numerical methods, Bifurcations, Saddle node, transcritical and pitchfork, Flows on the circle, Uniform and non- uniform oscillator, Two dimensional systems, Beyonds linear systems, Phase portraits, topological consequences, fixed points and linearization, Conservative versus dissipative systems, Reversible systems, Limits cycles in non-conservative systems, Chaos, Lorentz system of equations, Fractals.

Recommended Books:

- Fletcher, R. (1980). *Practical Methods Of Optimization: Vol. 1 Unconstrained Optimization*. John Wiley & Sons.
- Diwekar, U. M. (2020). *Introduction to applied optimization* (Vol. 22). Springer Nature.
- Pillo, G., & Giannessi, F. (Eds.). (2013). *Nonlinear optimization and Applications*. Springer Science & Business Media.